جامعة بنها كلية الهندسة ببنها قسم الهندسة الكهربية
نموذج الإجابة امتحان مادة هندسة التحكم ك١٢٣٦ يوم ١٦/٦/ ٢٠١٤
مدرس بالقسم شوقي حامد عرفه

Benha University	Time: 3-hours
Benha Faculty of Engineering	Second Year 2013/2014
Control Engineering (E1236)	Elect.Eng.Dept.

Solve as much as you can questions in two pages

Q1 **(20marks)**

a- Write a **mathematical model** represents the physical systems shown in Fig.1, and Fig.2?

b- Find the unity feedback control system represents the system shown in Fig.1 as R=1Ω, L= (1/6)H, C=(1/6)F using **block reduction method**?

c- Write the most important features of **good** control system?

d- Write the most important **advantages and disadvantages** of the **open** loop and the **closed** loop control systems?

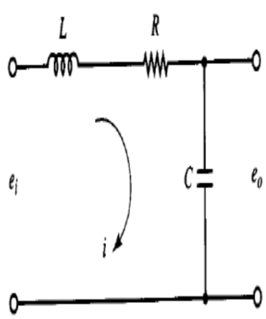


Fig.1

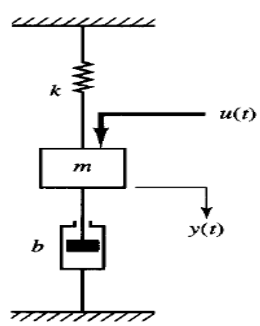


Fig.2

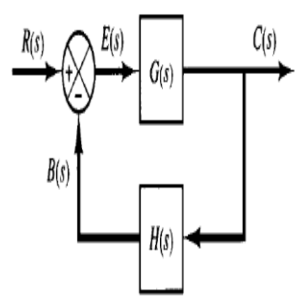


Fig.3

Q2 **(20 marks)**

Consider a system shown in Fig. 3 $H(s) = 1, \quad G(s) = \frac{K}{S(S+2)}$

a- Find the steady state static **error coefficients**?

b- Find the gain **K** such that the steady state error =0.02?

c- Find and draw the **unit step response** as K=4?

d- Find the **frequency response** and **M_r and ω_r** as K=4 and r(t)=5 sin ωt ?

P.T.O

Q3 **(25 marks)**

Consider a system shown in Fig. 3

a- Prove that the gain margin=∞ **db at** ∞**rad/sec**. and the phase margin= **51.8 degrees at 2.36 rad/sec**.?
b- Sketch the **polar plot** as K=9?
c- Sketch the **Bode plot** as K=9**?**
d- Sketch the **Nichols plot** as K=9**?**
e- Write short MATLAB program to solve a, b, c, and d?

Q4 **(10 marks)**

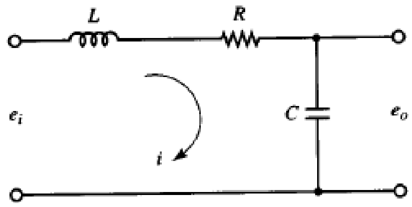
Consider a system shown in Fig.3

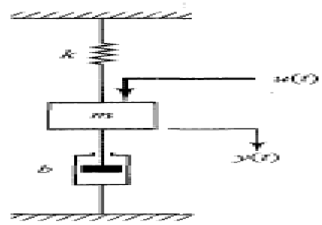
a- Sketch the **complete root locus** for positive values of **K**?
b- Using the **root locus** plot to find **K** as a damping ratio =**0.7**?
c- Write short MATLAB program to solve a?

Answer

1- Write a **mathematical model** represents the physical systems shown in Fig.1, and Fig.2?

The algebraic sum of all voltages around a closed loop in an electrical circuit at any given instant is zero





Newton's laws for mechanical systems:

$$ma = \sum F = m\ddot{X} = F - b\dot{X} + KX, \ \sum T = J\ddot{\theta} = T - b\dot{\theta} - K\theta$$

Where: m=mass in Kg, a=acceleration in m/sec^2, F=force in newtons

b-Find the unity feedback control system represents the system shown in Fig.1 as R=1Ω, L= (1/6)H, C=(1/6)F using **block reduction method**?

$$\frac{E_o(s)}{E_i(s)} = \frac{G(s)}{1 + G(s)H(s)} = \frac{36}{S^2 + 6S + 36}$$

$$then G(s) = \frac{36}{S(S+6)}, \mathrm{H(s)} = 1$$

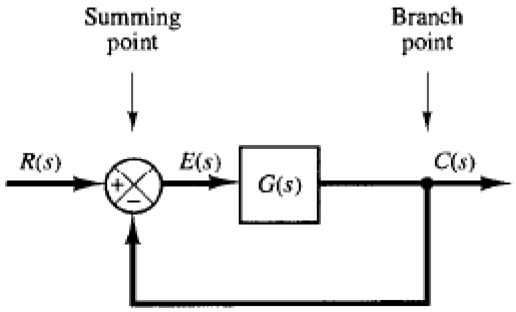


Figure 3–5
Block diagram of a closed-loop system.

$$\frac{E_o(s)}{E_i(s)} = \frac{1}{LCs^2 + RCs + 1}$$

$$G(s) = \frac{1/LC}{S(S+R/L)} = \frac{36}{S(S+6)}$$

c-Write the most important features of **good** control system?

1-simple construction and operation 2-fast response (speed) 3-less cost

4- large accuracy (less error) 5-stable

d-Write the most important **advantages and disadvantages** of the **open** loop and the **closed** loop control systems?

Open loop control system

Advantages of open loop	disadvantages of open loop
1-simple construction	1-disturbances cause errors
2- ease of maintenance	2-changes in calibration cause errors
3-less expensive	3-recalibration is necessary
4-no stability problem	
5-convenient when output is hard to measured or economically not feasible	

Closed loop control system

Disadvantages of closed loop	advantages of closed loop
1-complex construction	1-disturbances do not cause errors
2- stability may be a problem	2- has less errors
3-more expensive	3-recalibration is not necessary
	4-the ability to adjust the response

<u>Q2</u> **(20 marks)**

Consider a system shown in Fig. 3 $H(s) = 1, \quad G(s) = \frac{K}{S(S+2)}$

a- Find the steady state static **error coefficients**?

$$K_p = \lim_0 G(S) = \lim_{0} \frac{K}{S(S+2)} = \frac{K}{(0)(0+2)} = \infty$$

$K_v = \lim_0 SG(S) = \lim_0 \frac{K}{(S+2)} = \frac{K}{(0+2)} = 0.5K$

$K_a = \lim_0 S^2 G(S) = \lim_0 \frac{S\,K}{(S+2)}$ =0

b- Find the gain **K** such that the ramp steady state error =0.02?

$e_{ss}(t) = 1/K_v = \frac{1}{0.5K} = 0.02,\ K =$ 100

Routh test as K=100, system is stable

c- Find and draw the **unit step response** as K=4? $H(s) = 1, \quad G(s) = \frac{K}{S(S+2)}$

Step response of a second order system R(S)=1/s

C(S) =closed loop T.F(S)*R(S) = $\frac{\omega_n{}^2 R(S)}{S(\,S^2+2\eta\omega_n S+\omega_n{}^2)} = \frac{4}{S(S^2+2S+4)}$

$= \frac{a}{S} + \frac{bs+d}{(S^2+2S+4)}\ partial\ fraction$, Type equation here.

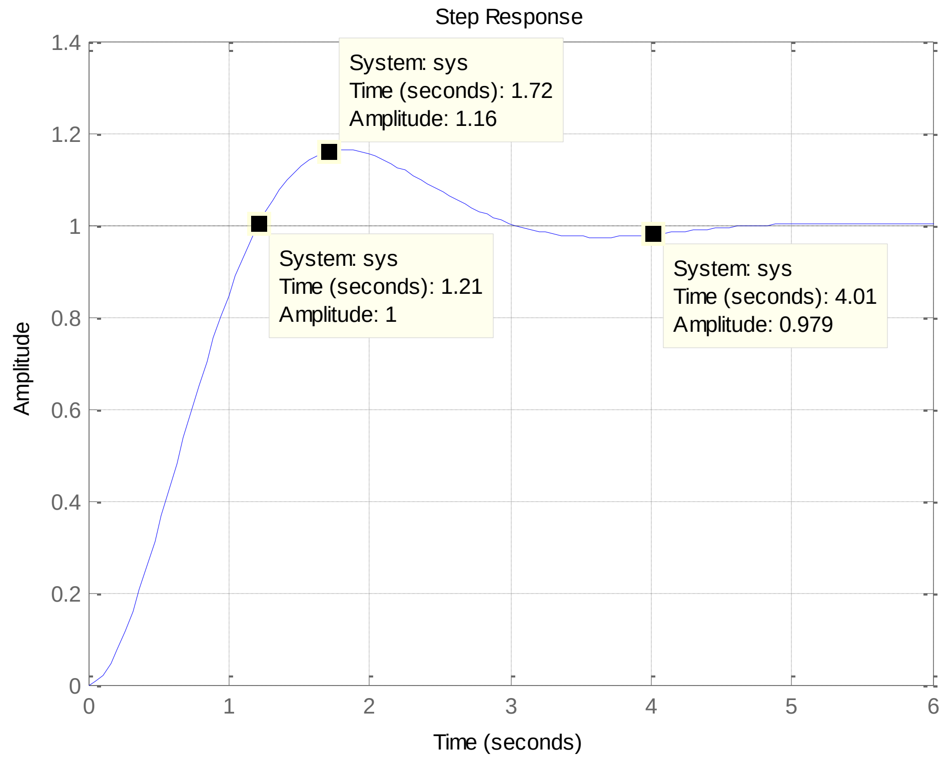
C(t)= inverse Laplace of the product of closed loop t.f.(S) and R(S)=1/s with zero initial conditions C(t)= L^{-1}[(C(S)]=L^{-1}[closed loop t.f.(S)*R(S)] with zero initial conditions]

$\eta = 0.5, \omega_n$

$C(t) = 1 - \frac{e^{-}}{\sqrt{}}$

$M_{p=e}^{-\frac{\eta\pi}{\sqrt{1-\eta^2}}} = 0.163, t_r = \frac{\pi - cos^{-1}\eta}{\omega_d} = \frac{\pi - \frac{pi}{3}}{1.732} = 1.21sec,$

$t_p = \frac{\pi}{\omega_d} = 1.81sec., t_s = 4T = \frac{4}{\eta\omega_n} = 4sec.$



d- Find the **frequency response** and $\mathbf{M_r}$ **and** $\boldsymbol{\omega_r}$ as K=4 and r(t)=5 sin ωt ?

$$\boldsymbol{\omega_r = \omega_n\sqrt{1-2\zeta^2}} = \mathbf{2\sqrt{1-2(0.5)^2}} = \frac{1.414rad}{sec}.$$

$$\boldsymbol{M_r} = \frac{\mathbf{1}}{\mathbf{2\zeta\sqrt{1 -}}}$$

-Steps to find frequency Response:

1-the closed loop transfer function =**T(s)=C(S)/R(S)** =

$$C(S) / R(S) = \frac{G(s)}{1+G(S)H(S)} = \frac{\omega_n{}^2}{S^2+2\eta\omega_n S+\omega_n{}^2} = \frac{4}{S^2+2S+4},\ \omega_n = \frac{2rad}{sec}\ \zeta = 0.5$$

2-the closed loop frequency transfer function =

$$\mathbf{T\,(j\omega)=C(j\omega)/R(j\omega)} = \frac{4}{(j\omega)^2+2(j\omega)+4} = M\llcorner\Phi = Re+j\ imag$$

$$\boldsymbol{M} = \frac{\mathbf{4}}{\sqrt{(\mathbf{4}-\boldsymbol{\omega}^2)^2+\mathbf{4}\boldsymbol{\omega}^2}}\quad ,\ \boldsymbol{\Phi} = -\mathbf{tan^{-1}[2\omega\,/(4-\omega^2)]}$$

3-As the input $= r(t) = 5 sin\omega t$ then

the response = $C(t) = 5M sin(\omega t + \Phi)$

$$= \frac{\mathbf{20}}{\sqrt{(\mathbf{4} - \boldsymbol{\omega}^2)^2 + \mathbf{4}\boldsymbol{\omega}^2}} \sin[\omega t - \mathbf{tan}^{-1}[\mathbf{2}\boldsymbol{\omega} /(\mathbf{4} - \boldsymbol{\omega}^2)]]$$

<u>Q3</u> **(25 marks)**

Consider a system shown in Fig. 3

a- Prove that the gain margin=∞ **db at** ∞**rad/sec**. and the phase margin= **51.8 degrees at 2.36 rad/sec**.?

$$\mathbf{G(j\omega)H(j\omega)} = \frac{\mathbf{9}}{\mathbf{j\omega[(j\omega+3)]}} = \mathrm{M}e^{j\Phi} = M\llcorner\Phi = \mathrm{Re+j\ imag}$$

$$\frac{\mathbf{9}}{\mathbf{j\omega[(j\omega+3)]}} = \frac{\mathbf{9}}{-\boldsymbol{\omega}^2 + \mathbf{j3\omega}} = \frac{\mathbf{9(-\boldsymbol{\omega}^2 - j3\omega)}}{(\boldsymbol{\omega}^4 + \mathbf{9}\boldsymbol{\omega}^2)} = \frac{-\mathbf{9}}{(\boldsymbol{\omega}^2 + \mathbf{9})} - \frac{\mathbf{27j\omega}}{(\boldsymbol{\omega}^4 + \mathbf{9}\boldsymbol{\omega}^2)}$$

$$\mathbf{Real} = \frac{-\mathbf{9}}{(\boldsymbol{\omega}^2 + \mathbf{9})} \approx -1$$

$$\boldsymbol{M} = \frac{\mathbf{9}}{\boldsymbol{\omega}\sqrt{\mathbf{9} + \boldsymbol{\omega}^2}} \quad , \ \boldsymbol{\Phi} = -\mathbf{90} - \mathbf{tan}^{-1}(\boldsymbol{\omega}/\mathbf{3})]$$

$$\boldsymbol{M}_{\boldsymbol{\omega}_g} = \frac{\mathbf{9}}{\boldsymbol{\omega}_g\sqrt{\mathbf{9} + \boldsymbol{\omega}_g^{\ 2}}} = 1\ , \boldsymbol{\omega}_g = \mathbf{2.36 rad/sec}$$

$$\boldsymbol{M}_{\boldsymbol{\omega}_p} = \frac{\mathbf{9}}{\boldsymbol{\omega}_p\sqrt{\mathbf{9} + \boldsymbol{\omega}_p^{\ 2}}} = \mathbf{0} \qquad G_m = 20\log(1/0) = \infty \mathrm{db}$$

$$\boldsymbol{\Phi}_{\boldsymbol{\omega}_p} = -\mathbf{tan}^{-1}(\boldsymbol{\omega}_p) - \mathbf{tan}^{-1}(\boldsymbol{\omega}_p/\mathbf{3}) = -180 \text{ deg. } \boldsymbol{\omega}_p = \frac{\infty rad}{sec}.$$

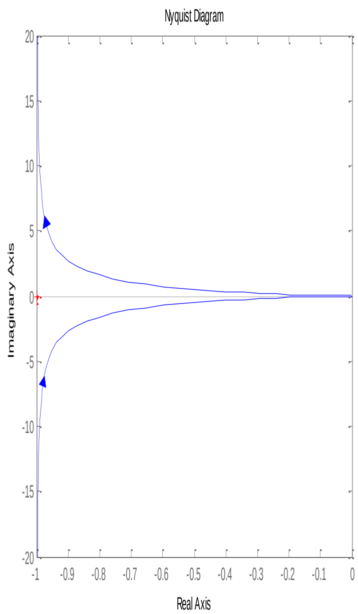
$$\boldsymbol{\Phi}_{\boldsymbol{\omega}_g} = -\mathbf{90} - \mathbf{tan}^{-1}(\boldsymbol{\omega}_g/\mathbf{3})] = -\mathbf{90} - \mathbf{38} = -\ \mathbf{128\ deg.}$$

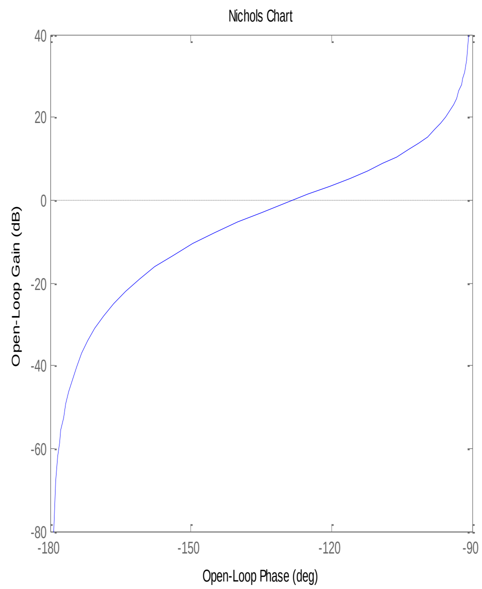
$$\gamma_m = \angle G(j\,\omega_g)H(j\,\omega_g) + 180 \text{ deg.} = 180 - 128 = 51.8 deg.$$

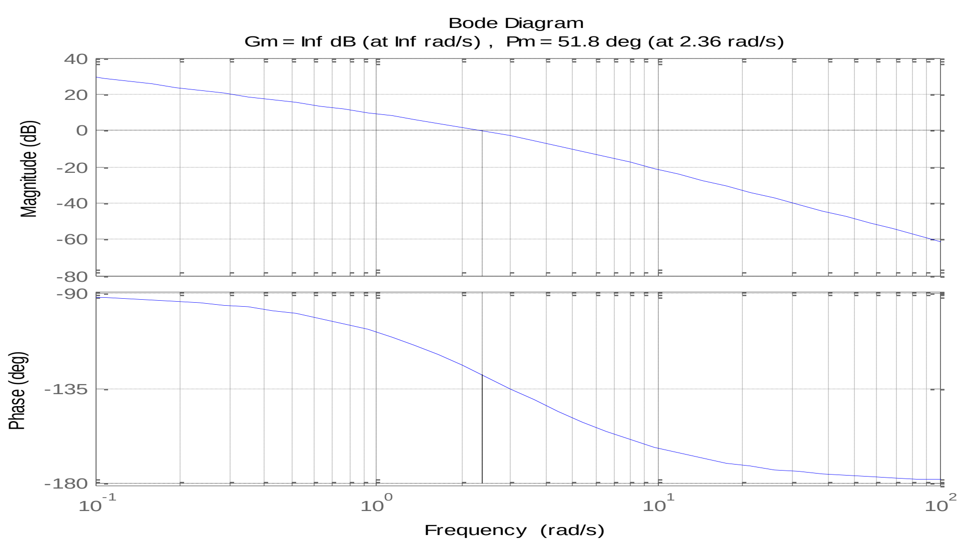
open loop TF=G(s) H(s)= **G(S) H(S) =9/[S(S+3)]**$=9/[S^2+3S+0]$

Find the table

ω	0	0.1	1	2.36	3	5	10	∞
Φ	-90	-92	-108.4	-128	-135	-149	-163.3	-180
M	∞	30	2.85	1	0.70	0.3	0.09	0
20logM		29.50	9.1	0	-3.1	-10.21	-21.31	







Prog. >>n=[9]; d=[1 3 0];

>> nyquist(n,d) >> margin(n,d) >> nichols(n,d)

Q4 **(10 marks)**

Consider a system shown in Fig.3

b- Sketch the **complete root locus** for positive values of **K**?
c- Using the **root locus** plot to find **K** as a damping ratio =**0.7**?
d- Write short MATLAB program to solve a?

Root locus:

1-the root locus is symmetrical about the real axis in the S-plane

2-the open loop TF=G(s) H(s)= **G(S) H(S) =K/[S(S+2.5)(S+1.5)]**=K/[S^3+4S^2+3.75S]

3-the root locus starts at the pole and ends at the zero or infinity

4-number of root loci= n=number of poles of the open loop TF =3 at [-1.5,-2.5,0]

5-number of zeros= m=0

6-number of asymptotes = n-m=3-0=3

8-center of gravity = $A = \frac{\sum poles - \sum zoles}{n-m} = \frac{-1-2-3}{3} = -2$point of intersection of asymptotes with real axis=

9-angles of asymptotes are = $\Theta = \frac{\pm 180\,(2R+1)}{n-m} = \pm 60, \pm 180$

10- Points of crossing the imaginary axis as Routh test

Charct.equa=1+G(S)H(S)=0= $S^3+6S^2+11s+6+K$

S^3	1	3.75	15 ≥ K ,K≥0 then 0≤K≤15,Kc=15 $S^3+4S^2+3.75S+K=0$, S=jω
S^2	4	K	
S	[15-K]/ 4		

S^0	K		$\omega=\sqrt{3.75}$ =1.94 rad/sec

11- break points (break away or break in) at

$$-\frac{dK}{dS}=0=\frac{d}{dS}\left[\frac{1}{G(S)H(S)}\right]=\frac{d}{dS}[S3+4S2+3.75S]=3S^2\ +8s+3.75=0$$

S=-2.1 refused, S=-0.6 is a break- away point

12-break angles at [±180(2R+1)/r] where r=number of branches(poles for break away or zeros for break in) R=0,1,----- break angles at [±180]/2=±90

13-there is no angle of departure (complex poles)

14- there is no angle of arrival (complex zeros)

15-sketch the root loci as

16- the damping factor or coefficient ζ is straight line with slope ϴ= $\cos^{-1}\zeta$

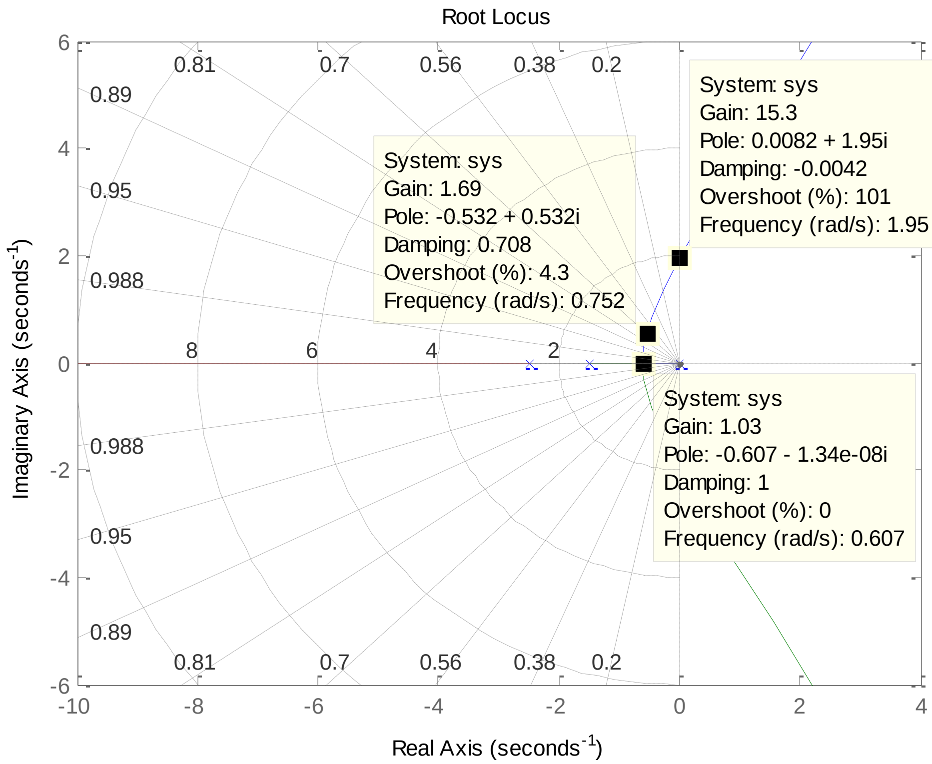
with respect to the negative real axis in the S-plane. ϴ= $\cos^{-1} 0.7$ =46 deg. at the test point (intersection point) S_d=-1.13±j1.13

angle conditio

magnitude c

$$\sum_{n=1}^{n=3}$$

$$\sum_{n=1}^{n=3} open\ loo$$



19- To find analytically closed loop poles and K as

$(S^2+2\,\zeta\,\omega_n\,S+\omega_n^2)(S+a)$=characteristic equa. for a third order syst.

Solve $1+G(S)H(S)=0=S^3+4S^2+3.75S+K=(S^2+1.4\,\omega_n\,S+\omega_n^2)(S+a)$

$=S^3+(1.4\,\omega_n+a)S^2+(1.4\,\omega_n\,a+\omega_n^2)S+\omega_n^2\,a$

$1.4\,\omega_n+a=6$,,,, $1.4\,\omega_n\,a+\omega_n^2=11$,,,, $\omega_n^2\,a=k+6$

Prog. >>n=[1];d=[1 4 3.75 0]; rlocus(n,d), grid