Benha University **College of Engineering at Banha**
Department of Mechanical Eng.

Subject: Fluid Mechanics

Model Answer of the Final Corrective Exam

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اجابة امتحان ميكانيكا الموائع م 1112 السنة الأولى ميكانيكا (نظام جديد)

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- **1-a** Recall from thermo class, that a **system** is defined as a volume of mass of fixed identity.
- **Conservation of mass** states that the mass of a system is constant.

This can be written as the following equation:

$$\frac{dm_{sys}}{dt} = 0$$

Conservation of linear momentum

which is a restatement of Newton's Second Law.

Newton's Second Law

- In equation form this is written as: $\Sigma \underline{F}_{sys} = \frac{d}{dt}(m\underline{V})_{sys}$

Where $m\underline{V}$ = the linear momentum of the system.

Conservation of Energy

- For this, use the First Law of Thermodynamics in rate form to obtain the following equation:

$$\frac{dE_{sys}}{dt} = \dot{Q}_{sys} - \dot{W}_{sys}$$

- Where E = the total energy of the system. In the above equation

$$\frac{dE_{sys}}{dt}$$

is the rate of change of system energy.

- $\dot{Q}_{sys}$ is the rate of heat added ***to*** the system $\left(\frac{\delta Q}{dt}\right)$
- $\dot{W}_{sys}$ is the rate of work done ***by*** the system $\left(\frac{\delta W}{dt}\right)$.

Because work is done by the system, the negative sign is in the equation for the first law of thermodynamics.

- Now, these conservation laws must always hold ***<u>for a system</u>***.

<u>Conservation of Angular Momentum</u>
We will have time to study this

1-b

- <u>Solution:</u> first, draw a C.V. inside the entire tank.

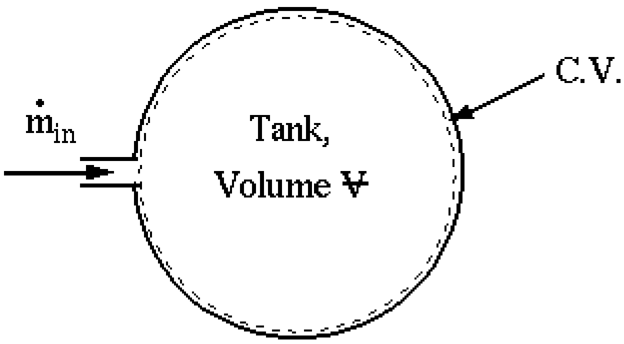


Figure 1

Now use our conservation of mass equation.

$$0 = \frac{d}{dt}\int_{CV} \rho \cdot dV + \sum_{outlets} \dot{m} - \sum_{inlets} \dot{m}$$

assume density is equal everywhere in the tank, and only varies with time. There are no outlets and the only term remaining is the mass term at the inlet:

$$0 = \frac{d}{dt}\int_{CV} \rho \cdot dV - \dot{m}_{in}$$

This is a differential equation we must solve by separating the variables and integrating from t=0 to some arbitrary t.

$$p = p_0 + \frac{\dot{m}_{in}}{Volume} \cdot t$$

Finally, use the ideal gas law to get the pressure. Thus,

$$p = p_0 + \frac{\dot{m}_{in}}{Vol.} \cdot tRT$$

(NOTE: pressure varies linearly with t)

2-a)

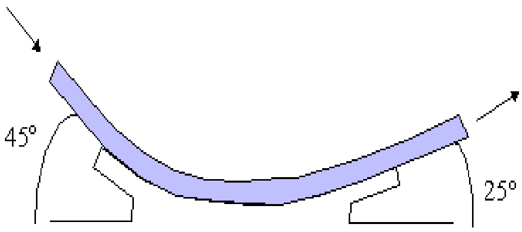


Figure 2

From the question:

$$\begin{aligned} a_1 &= 0.075 \times 0.025 = 1.875 \times 10^{-3}\ m^2 \\ u_1 &= 25\ m/s \\ Q &= 1.875 \times 10^{-3} \times 25\ m^3/s \\ a_1 &= a_2, \qquad so \qquad u_1 = u_2 \end{aligned}$$

Calculate the total force using the momentum equation:

$$\begin{aligned} F_{T_x} &= \rho Q(u_2 \cos 25 - u_1 \cos 45) \\ &= 1000 \times 0.0469(25\cos 25 - 25\cos 45) \\ &= 233.44\ N \end{aligned}$$

$$\begin{aligned} F_{T_y} &= \rho Q(u_2 \sin 25 - u_1 \sin 45) \\ &= 1000 \times 0.0469(25\sin 25 - 25\sin 45) \\ &= 1324.6\ N \end{aligned}$$

Body force and pressure force are 0. , So force on vane:

$$\begin{aligned} R_x &= -F_{t_x} = -233.44N \\ R_y &= -F_{t_y} = -1324.6N \end{aligned}$$

2-b)

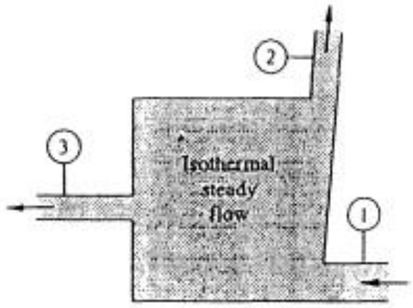


Figure 3

For continuity, $Q_3 = Q_1 - Q_2 = 120$ m3/hr. Establish the velocities at each port:

$$V_1 = \frac{Q_1}{A_1} = \frac{220/3600}{\pi(0.045)^2} = 9.61\ \frac{m}{s};\quad V_2 = \frac{100/3600}{\pi(0.035)^2} = 7.22\ \frac{m}{s};\quad V_3 = \frac{120/3600}{\pi(0.02)^2} = 26.5\ \frac{m}{s}$$

With gravity and heat transfer and internal energy neglected, the energy equation becomes

$$\dot{Q} - \dot{W}_s - \dot{W}_v = \dot{m}_3\left(\frac{p_3}{\rho_3} + \frac{V_3^2}{2}\right) + \dot{m}_2\left(\frac{p_2}{\rho_2} + \frac{V_2^2}{2}\right) - \dot{m}_1\left(\frac{p_1}{\rho_1} + \frac{V_1^2}{2}\right),$$

or: $$-\dot{W}_s/\rho = \frac{100}{3600}\left[\frac{225000}{998} + \frac{(7.22)^2}{2}\right] + \frac{120}{3600}\left[\frac{265000}{998} + \frac{(26.5)^2}{2}\right]$$

$$+ \frac{220}{3600}\left[\frac{150000}{998} + \frac{(9.61)^2}{2}\right]$$

Solve for the shaft work: $\dot{W}_s$ = 998(–6.99 –20.56 +12.00) H =**15500 W** *Ans.*
(negative denotes work done *on* the fluid)

3-a)

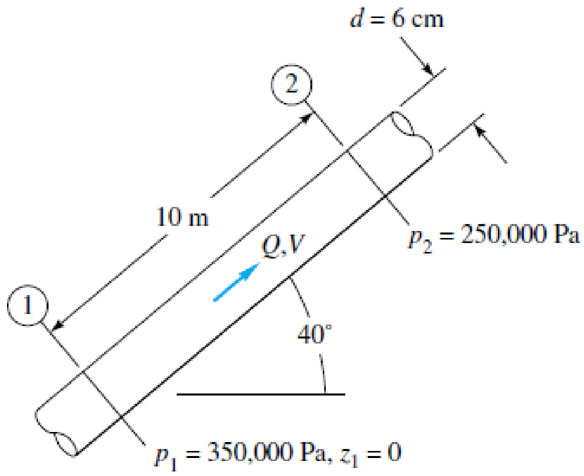


Figure 4

Part (a) For later use, calculate

$$\mu = \rho\nu = (900\ \text{kg/m}^3)(0.0002\ \text{m}^2/\text{s}) = 0.18\ \text{kg/(m} \cdot \text{s)}$$

$$z_2 = \Delta L \sin 40° = (10\ \text{m})(0.643) = 6.43\ \text{m}$$

The flow goes in the direction of falling HGL; therefore compute the hydraulic grade-line height at each section

$$\text{HGL}_1 = z_1 + \frac{p_1}{\rho g} = 0 + \frac{350{,}000}{900(9.807)} = 39.65 \text{ m}$$

$$\text{HGL}_2 = z_2 + \frac{p_2}{\rho g} = 6.43 + \frac{250{,}000}{900(9.807)} = 34.75 \text{ m}$$

The HGL is lower at section 2; hence the flow is from 1 to 2 as assumed. *Ans. (a)*

Part (b) The head loss is the change in HGL:

$$h_f = \text{HGL}_1 - \text{HGL}_2 = 39.65 \text{ m} - 34.75 \text{ m} = 4.9 \text{ m}$$ *Ans. (b)*

Half the length of the pipe is quite a large head loss.

Part (c) We can compute Q from the various laminar-flow formulas, notably

$$Q = \frac{\pi \rho g d^4 h_f}{128 \mu L} = \frac{\pi(900)(9.807)(0.06)^4(4.9)}{128(0.18)(10)} = 0.0076 \text{ m}^3/\text{s}$$ *Ans. (c)*

Part (d) Divide Q by the pipe area to get the average velocity

$$V = \frac{Q}{\pi R^2} = \frac{0.0076}{\pi(0.03)^2} = 2.7 \text{ m/s}$$ *Ans. (d)*

Part (e) With V known, the Reynolds number is

$$\text{Re}_d = \frac{Vd}{\nu} = \frac{2.7(0.06)}{0.0002} = 810$$ *Ans. (e)*

3-b

i) This region, where there is a velocity profile in the flow due to the shear stress at the wall, we call the **boundary layer**.

Boundary layer is the region near a solid where the fluid motion is affected by the solid boundary.

ii) Once the boundary layer has reached the centre of the pipe the flow is said to be **fully developed**. (Note that at this point the whole of the fluid is now affected by the boundary friction.)

At a finite distance from the entrance, the boundary layers merge and the inviscid core disappears. The flow is then entirely viscous, and the axial velocity adjusts slightly further until at $x = L_e$ it no longer changes with x and is said to be fully developed, $v = v(r)$ only.

iii) The length of pipe before fully developed flow is achieved is different for the two types of flow. The length is known as the **entry length**.

The entrance length $\mathbf{L_e}$ is estimated for laminar flow to be :

$L_e/D = 0.06\, Re_D$ for laminar

$L_e/D = 4.4\, Re_D^{1/6}$ for turbulent flow

Where L_e is the entrance length; and

Re_D is the Reynolds number based on Diameter

4-a)

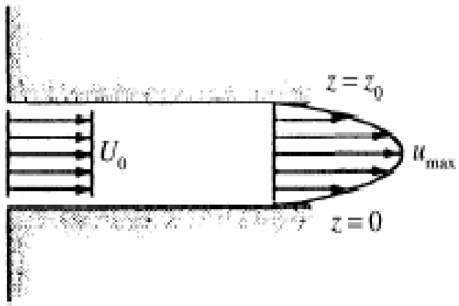


FIGURE 5

The flow rate per unit width of the area $Q = U_o z_0 * 1 = 8 \times 4 \times 1 = 32 cm^3/\sec$

$$Q = \int_0^{z_o} az(z_0 - z)dz = a(\frac{z_0^3}{2} - \frac{z_0^3}{3}) = a\frac{z_0^3}{6}$$

$$\therefore a\frac{4^3}{6} = 32 \Rightarrow a = \frac{6}{2} = 3$$

U_{max} at the middle where $z = \frac{z_o}{2} = 2cm \Rightarrow u_{max} = 3 \times 2(4-2)$ =12 cm/sec (i)

The shear stress $\tau = \mu\frac{du}{dz}$ at the wall i.e z=0 $\therefore \tau = \mu\frac{du}{dz} = \mu a z_0$ =0.29x3x4=3.48 \n/m^2

The skin friction coefficient

$$C_F = \frac{\tau}{\frac{1}{2}\rho U_o^2} = \frac{3.48}{0.5 \times 891 \times (0.08)^2} = 1.2205 \qquad \text{(ii)}$$

$$\tau dx = dp \times z_o \Rightarrow \frac{dp}{dx} = \frac{\tau}{z_o} = -87 Pa/m \qquad \text{(iii)}$$

4-b)

The displacement thickness is given by

$$\delta^* = \int_0^\infty \left(1 - \frac{v_x}{U}\right) dy = \int_0^\delta \left(1 - \left(\frac{y}{\delta}\right)^{1/7}\right) dy$$

$$= \delta - \frac{7\delta}{8} = \frac{\delta}{8} \cong 0.125\ \delta$$

and the momentum thickness is given by

$$\theta = \int_0^\infty \frac{v_x}{U}\left(1 - \frac{v_x}{U}\right) dy = \int_0^\delta \left(\frac{y}{\delta}\right)^{1/7}\left(1 - \left(\frac{y}{\delta}\right)^{1/7}\right) dy$$

$$= \frac{7\delta}{8} - \frac{7\delta}{9} = \delta\left(\frac{7}{8} - \frac{7}{9}\right) = \frac{7\delta}{72} \cong 0.0972\ \delta$$

Thus, the shape factor is

$$H = \frac{\delta^*}{\theta} = \frac{\frac{\delta}{8}}{\frac{7\delta}{72}} = \frac{9}{7} \cong 1.29$$

4-c)

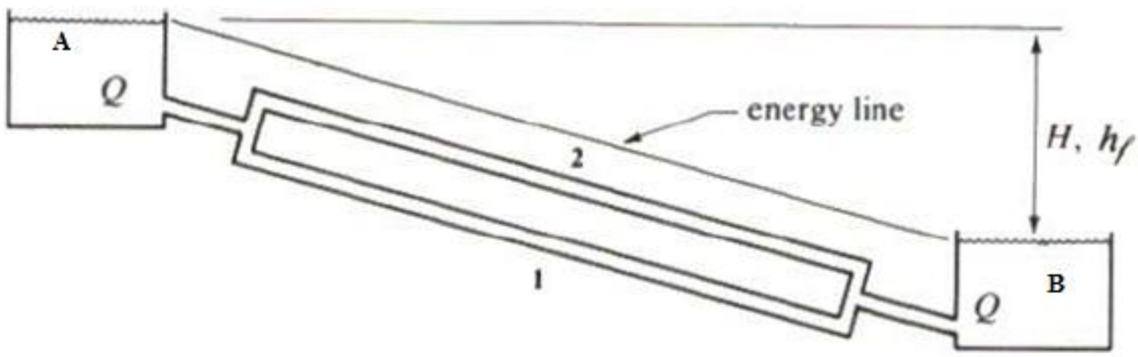


Figure 6

Apply Bernoulli to each pipe separately. For pipe 1:

$$\frac{p_A}{\rho g} + \frac{u_A^2}{2g} + z_A = \frac{p_B}{\rho g} + \frac{u_B^2}{2g} + z_B + 0.5\frac{u_1^2}{2g} + \frac{4flu_1^2}{2gd_1} + 1.0\frac{u_1^2}{2g}$$

p_A and p_B are atmospheric, and as the reservoir surface move s slowly u_A and u_B are negligible, so

$$z_A - z_B = \left(0.5 + \frac{4fl}{d_1} + 1.0\right)\frac{u_1^2}{2g}$$

$$10 = \left(1.0 + \frac{4 \times 0.008 \times 100}{0.05}\right)\frac{u_1^2}{2 \times 9.81}$$

$$u_1 = 1.731 m/s$$

And flow rate is given by:

$$Q_1 = u_1 \frac{\pi d_1^2}{4} = 0.0034\, m^3/s$$

For pipe 2:

$$\frac{p_A}{\rho g} + \frac{u_A^2}{2g} + z_A = \frac{p_B}{\rho g} + \frac{u_B^2}{2g} + z_B + 0.5\frac{u_2^2}{2g} + \frac{4flu_2^2}{2gd_2} + 1.0\frac{u_2^2}{2g}$$

Again p_A and p_B are atmospheric, and as the reservoir surface move s slowly u_A and u_B are negligible, so

$$z_A - z_B = \left(0.5 + \frac{4fl}{d_2} + 1.0\right)\frac{u_2^2}{2g}$$

$$10 = \left(1.0 + \frac{4 \times 0.008 \times 100}{0.1}\right)\frac{u_2^2}{2 \times 9.81}$$

$$u_2 = 2.42\, m/s$$

And flow rate is given by:

$$Q_2 = u_2 \frac{\pi d_2^2}{4} = 0.0190\, m^3/s$$

5-a)

i) For the function $P = \text{fcn}(D, \rho, V, \Omega, n)$ the appropriate dimensions are $\{P\} = \{ML^2T^{-3}\}$, $\{D\} = \{L\}$, $\{\rho\} = \{ML^{-3}\}$, $\{V\} = \{L/T\}$, $\{\Omega\} = \{T^{-1}\}$, and $\{n\} = \{1\}$. Using (D, ρ, V) as repeating variables, we obtain the desired dimensionless function:

$$\frac{P}{\rho D^2 V^3} = fcn\left(\frac{\Omega D}{V}, n\right) \quad Ans.\ (\text{i})$$

iii) "Geometrically similar" means that n is the same for both windmills. For "dynamic similarity," the advance ratio $(\Omega D/V)$ must be the same:

$$\left(\frac{\Omega D}{V}\right)_{model} = \frac{(4800\ r/\text{min})(0.5\ m)}{(40\ m/s)} = 1.0 = \left(\frac{\Omega D}{V}\right)_{proto} = \frac{\Omega_{proto}(5\ m)}{12\ m/s},$$

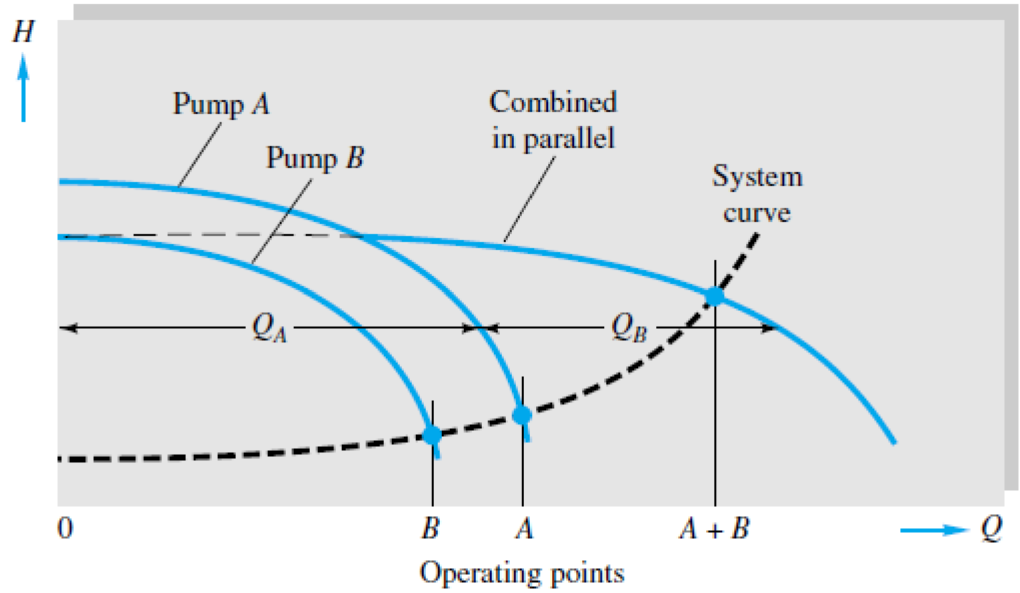
$$\text{or:} \quad \Omega_{proto} = \mathbf{144\ \frac{rev}{min}} \quad Ans.\ (\text{iii})$$

ii) At 2000 m altitude, $\rho = 1.0067\ kg/m^3$. At sea level, $\rho = 1.2255\ kg/m^3$. Since $\Omega D/V$ and n are the same, it follows that the power coefficients equal for model and prototype:

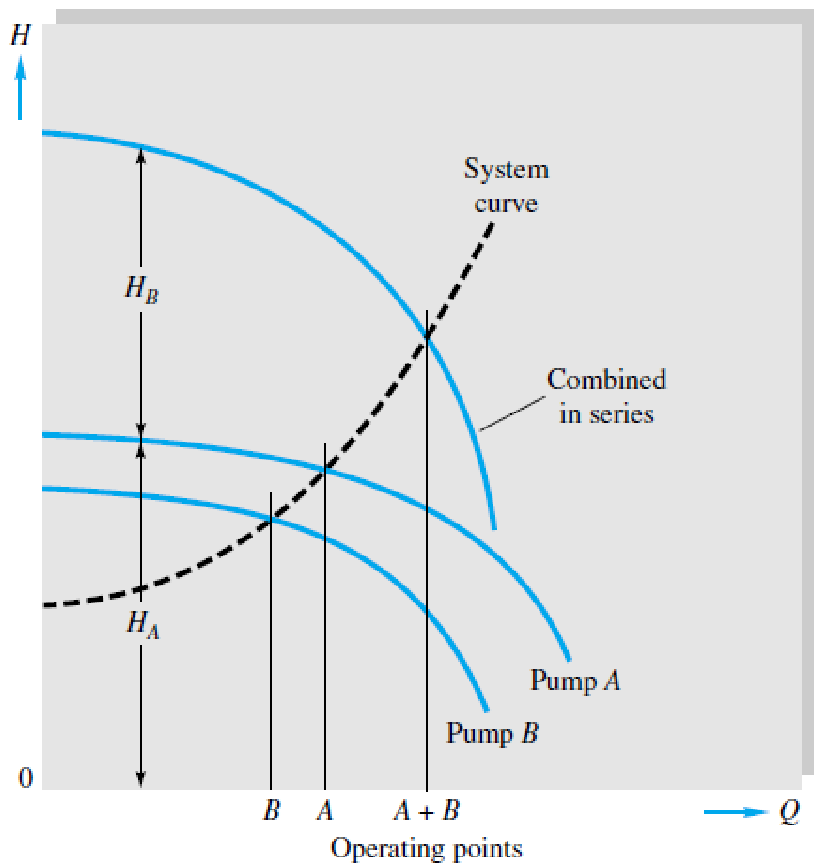
$$\frac{P}{\rho D^2 V^3} = \frac{2700W}{(1.2255)(0.5)^2(40)^3} = \frac{P_{proto}}{(1.0067)(5)^2(12)^3},$$

$$solve \quad \mathbf{P_{proto}} = 5990\ W \approx \mathbf{6\ kW} \quad Ans.\ (\text{ii})$$

5-b)



Performance and operating points of two pumps operating singly and combined in parallel



Performance and operating points of two pumps operating singly and combined in series