Benha University
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Model Answer of the Final Exam Date:1/6/2013

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اجابة امتحان ديناميكا الموائع والغازات (م401) 2 يونية 2013
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1-a**1-a)** .The equation relating the relative change in area with the relative change in velocity is given by:

$$\frac{dA}{A}=(M^2-1)\frac{dV}{V}$$

which may alternatively be written as:

$$\frac{dA}{dV}=(M^2-1)\frac{A}{V}$$

Because A and V are positive, it may be concluded from the above two equations that:

1. If $M<1$, i.e., if the flow is subsonic, then dA has the opposite sign to dV, i.e., decreasing the area increases the velocity and vice versa.
2. If $M>1$, i.e., if the flow is supersonic, then dA has the same sign as dV, i.e., decreasing the area decreases the velocity and vice versa.
3. If $M=1$ then $dA/dV=0$ and A reaches an extremum. From (1) and (2) it follows that when $M=1$, A must be a minimum.

b)

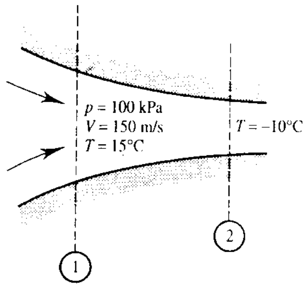

Fig.1

The absolute temperature at section 1 is T_1 = 273+15=288 K
The Mach number at section 1 is M_1 = 0.4409
The absolute temperature at section 2is T_2= 273-10 =263 K
From the energy equation

$$m/s\,T_1+\frac{v_1^2}{2}=C_pT_2+\frac{v_2^2}{2}\Rightarrow v_2=218$$

The Mach number at section 2 is M_2 =0.671

$$\rho_1=\frac{p_1}{RT_1}=1.209\ \text{kg/m}^3 \text{ from the equation } \frac{\rho_2}{\rho_1}=\left(\frac{T_2}{T_1}\right)^{\frac{1}{\gamma-1}} = 0.7969$$

$$\rightarrow\therefore\rho_2=0.9635\ \text{kg/m}^3$$

From the continuity equation $\rho_1v_1A_1=\rho_2v_2A_2\rightarrow\frac{A_2}{A_1}=0.8634$

c) $\rho_1 = \dfrac{p_1}{RT_1} = 0.4598$, V_1= 872.5 m/s

the mass flow rate $\dot{m} = \rho_1 v_1 A_1 = 0.9628$ kg/sec.

For M_1 = 2.5

$M_1 = 2.5 \rightarrow \dfrac{A_1}{A_1^*} = 2.6367$, $\dfrac{p_1}{p_{o1}} = 0.0585$ to obtain the Mach number before the shock M_{2n1} we get

$\dfrac{A_2}{A_1^*} = \dfrac{A_2}{A_1}\dfrac{A_1}{A_1^*} = 1.978$ $\rightarrow$ from table $M_{2n1} = 2.185$ $\rightarrow$ from the normal shock table we get the Mach number after the shock $M_{2n2} = 0.549$, also we get $\dfrac{P_{o2}}{P_{o1}} = 0.635$, we have $P_{o1} =$ 683,76 kPa

$\rightarrow \therefore P_{o2} =$ 414.2 kPa

From table A1 for $M_{2n2} = 0.549$ we get $\dfrac{A_2}{A_2^*} =$ 1.257

$\dfrac{A_3}{A_2^*} = \dfrac{A_3}{A_2}\dfrac{A_2}{A_2^*} =$ 2.235

The Mach number at section 3 is M_3 = 0.25

The stagnation pressure at section 3 is $P_{o3} = 414.2$ kPa

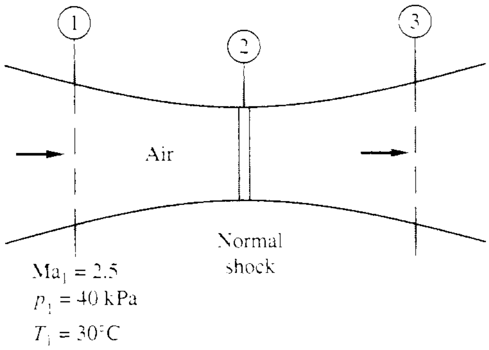

Fig.2

2-a) Rayleigh flow is one-dimensional frictionless flow in constant area duct with heat addition and removal

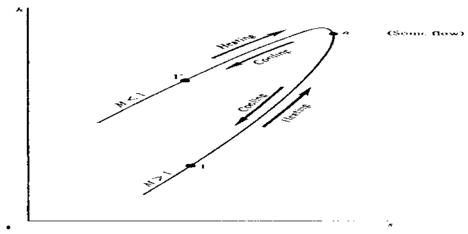

Figure3. Rayleigh curve

Heating always pushes the flow towards the sonic conditions and vice versa. This is clear on the *Rayleigh curve* shown above In detail we can conclude the following:

1. For *supersonic flow* in region 1, i.e., $M_1 > 1$, when heat is added
   a. Mach number decreases, $M_2 < M_1$
   b. Pressure increases, $p_2 > p_1$
   c. Temperature increases, $T_2 > T_1$
   d. Total temperature increases, $T_{o_2} > T_{o_1}$
   e. Total pressure decreases, $p_{o_2} < p_{o_1}$
   f. Velocity decreases, $u_2 < u_1$
2. For *subsonic* flow in region 1, i.e., $M_1 < 1$, when heat is added
   a. Mach number increases, $M_2 > M_1$
   b. Pressure decreases, $p_2 < p_1$
   c. Temperature increases for $M_1 < \gamma^{-1/2}$ and decreases for $M_1 > \gamma^{-1/2}$
   d. Total temperature increases, $T_{o_2} > T_{o_1}$
   e. Total pressure decreases, $p_{o_2} < p_{o_1}$
   f. Velocity increases, $u_2 > u_1$

For heat extraction (cooling of the flow), all of the above trends are opposite.

From the development here, it is important to note that heat addition always drives the Mach numbers toward 1, decelerating a supersonic flow and accelerating

2-b) The situation under consideration is shown in Fig.4

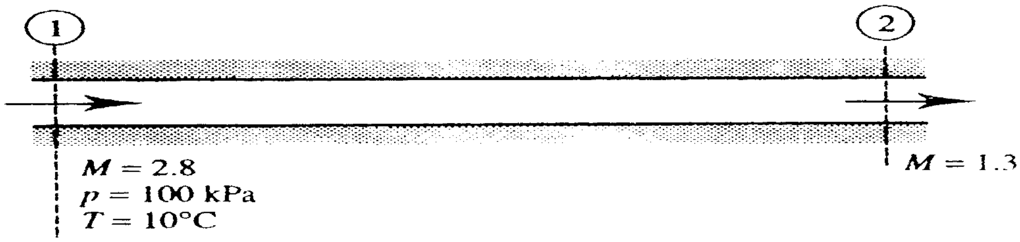

Fig.4

Now for Mach number 2.8 using the relation.

$$\frac{T_0}{T} = 1 + \frac{\gamma - 1}{2} M^2 = 1 + 0.2M^2$$

or using the software or isentropic tables gives $T_{01}/T_1 = 2.568$. Hence

$$T_{01} = 2.568 \times (273 + 10) = 726.7 \text{ K}$$

Next, using the relations given above or the tables or the software for frictionless flow in a constant area duct with heat exchange gives:

For $M = 2.8$:

$$\frac{p}{p^*} = 0.2004, \quad \frac{T}{T^*} = 0.3149, \quad \frac{T_0}{T_0^*} = 0.6738$$

For $M = 1.3$:

$$\frac{p}{p^*} = 0.7130, \quad \frac{T}{T^*} = 0.8592, \quad \frac{T_0}{T_0^*} = 0.9580$$

Using these values gives:

$$p_2 = \frac{p_2/p^*}{p_1/p^*} p_1 = \frac{0.7130}{0.2004} \times 100 = 355.8 \text{ kPa}$$

and:

$$T_2 = \frac{T_2/T^*}{T_1/T^*} T_1 = \frac{0.8592}{0.3149} \times 283 = 772.2 \text{ K} = 499.2°\text{C}$$

Therefore, the pressure and temperature of the air at the outlet to the duct are 355.8 kPa and 499.2°C respectively.

When the maximum amount of heat is transferred to the flow, the Mach number at exit becomes one, then the total temperature T_{02} will equal T_o^*.Hence in this case:

$$T_{02} = \frac{T_{01}}{T_{01}/T_0^*} = \frac{726.7}{0.6738} = 1078.5\text{K}$$

but $q = Cp(T_{02} - T_{01}) = 1.007(1078.5\text{-}726.7) = 354.3$ kJ/kg

it having been assumed that Cp = 1.007 kJ/kg.

Also when the maximum amount of heat has been transferred:

$$p_2 = p^* = \frac{p_1}{p_1/p^*} = \frac{100.7}{0.2004} = 499.0\ \text{kPa}$$

and: $$T_2 = T^* = \frac{T_1}{T_1/T^*} = \frac{283}{0.3149} = 898.7\ \text{K} = 625.7°\text{C}$$

Therefore, when the maximum amount of heat is transferred, the heat added is 354.3 kJ for every kilogram of air flowing through the duct and under these circumstances the pressure and temperature of the air at the outlet to the duct are 499.0 kPa and 625.7°C respectively.

3-a) Fanno flow is the one-dimensional adiabatic flow in a constant area duct with friction effect. The mach number variation for supersonic inlet flow with the duct length is shown below. When L = L^* Mach number reaches unity at exit ; but when L increases ther exist a normal shock wave in the duct which moves forward as L increases more and more; this is shown in Fig.4.

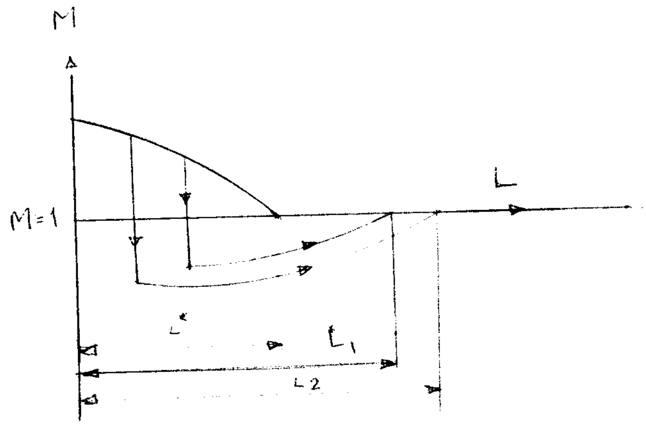

Figure 4

b) 1-The flow situation under consideration is shown in the figure 5 below

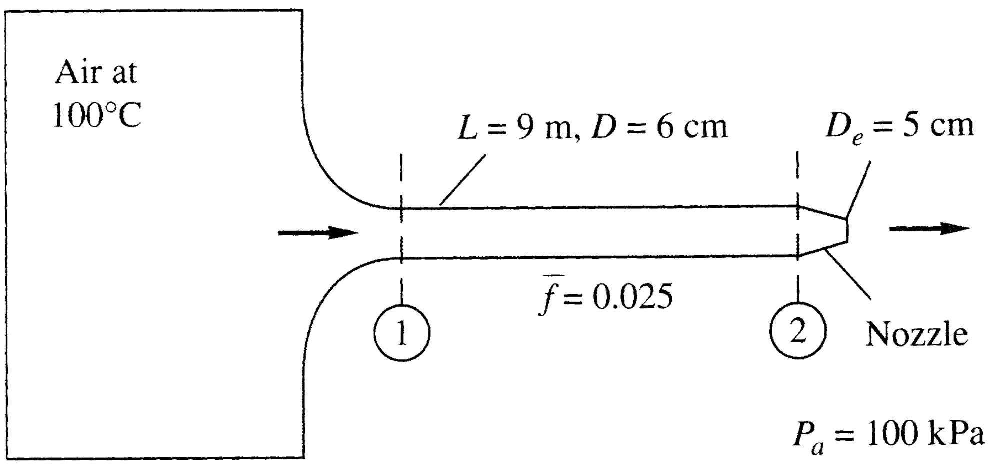

Figure 5

At the exit Mach number is 1 the area at exit is considered to the critical area so
$T_e = T^*$. Since $T_o = 100 + 273 = 373$ so $T_e = 310.8$ K

The exit speed $V_e = 353.5$ m/s

The mass flow rate $\dot{m} = \rho_e v_e A_e \rightarrow \rho_e = 1.44$ kg/s

$P_e = \rho_e R T_e \quad \therefore P_e = 146.2 \text{ kPa} = P^* \rightarrow P_{oe} = 276.75 \text{ kPa}$

The flow between section 2 and the exit is isentropic so $P_{oe} = P_{02}$ and $\frac{A_2}{A_e} = \frac{A_2}{A^*}$

$\frac{A_2}{A^*} = 1.44 \rightarrow$ from table $M_2 = 0.454$ and $\frac{p_2}{p_{o2}} = 0.868 \;\therefore p_2 = 240.2$ kPa

$\rightarrow$ from table and for $M_2 = 0.454$ we get $\frac{\bar{f}L_2^*}{D} = \frac{4fL_2^*}{D} = 1.57$, $\frac{p_2}{p^*} = 2.36$

$\frac{4fL_1^*}{D} - \frac{4fL_2^*}{D} = \frac{4f(L_1^* - L_2^*)}{D} = \frac{4f(9)}{D} = 3.75 \rightarrow \frac{4fL_1^*}{D} = 5.32$

$\rightarrow$ from table we get $M_1 = 0.3$ and $\frac{p_1}{p^*} = 3.6191$

$P_1 = (\frac{p_1}{p^*} / \frac{p_2}{p^*}) \times p_2 \;\rightarrow P_1 = 368.35$ kPa

The flow between the tank and section 1 is isentropic, so the pressure inside the tank equal to the stagnation pressure at section 1 $P_{tank} = P_{o1} = 392.07$ kPa

4-a) The assumptions for derivation of the linearzed velocity potential are :

i) The perturbations are very small $\frac{u}{U_\infty} << 1, \frac{v}{U_\infty} << 1, \frac{w}{U_\infty} << 1$.

ii) Ranges of Mach number are ,0< M_∞ <0.8, M_∞ >1.2

iii) Flow is not hypersonic M_∞ <5

These assumptions fail at leading edge where assumption (i) is invalid, for transonic flow for which .8< M_∞ <1.2 (assumption (ii) is invalid, and for hypersonic flow for which M_∞ >5.

b)The lower critical Mach number ($M_{\infty cr1}$) is the upstream (non-disturbed) flow Mach number at which the maximum local velocity on the airfoil reaches the speed of sound at one point only. This point is called the minimum pressure point or the point of maximum suction.
Evaluation the lower critical Mach number for airfoil with Cp_{imin}=-0.7 .
The characteristic Mach number at point of maximum suction when the airfoil is traveling at $M_{\infty cr1}$ is equal 1.0 i.e λ_c = 1.0, from Cristianowich diagram λi = 0.7577

From the equation $\lambda i = \lambda\infty i \sqrt{1 - Cp_i}$, $\lambda\infty i = \frac{0.7577}{\sqrt{1-(-0.7)}} = 0.581$

From Cristianowich diagram the characteristic Mach number for the upstream (non-disturbed) flow is $\lambda\infty c$ = 0.638 for which the $M_{\infty cr1}$ = 0.8118.

5-a) It is known that $\sin^{-1} 1/M_\infty < \beta < \pi/2$. Consequently $19.47^\circ < \beta < 90^\circ$.

b) The situation is shown below in Fig.6

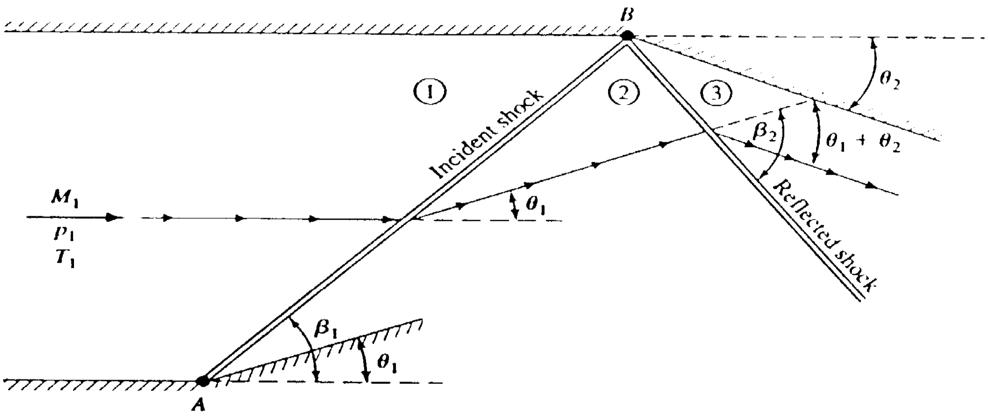

Figure 6

From the θ-β-M diagram, $\beta_1 = 31.2°$,

$$M_{n_1} = M_1 \sin\beta_1 = 3\sin 31.2° = 1.554$$

From Table A.2, for $M_{n_1} = 1.56$ (nearest entry),

$$\frac{p_2}{p_1} = 2.673, \quad \frac{T_2}{T_1} = 1.361, \quad M_{n_2} = 0.6809$$

$$M_2 = \frac{M_{n_2}}{\sin(\beta_1 - \theta_1)} = \frac{0.6809}{\sin(31.2 - 14)} = 2.30$$

The flow in region 2, at $M_2 = 2.3$, is deflected downward through the combined angle $\theta_1 + \theta_2 = 14° + 10° = 24°$. From the θ-β-M diagram for $M = 2.3$ and $\theta = 24°$, $\beta_2 = 52.5°$,

$$M_{n_2} = M_2 \sin\beta_2 = 2.3\sin 52.5° = 1.82$$

From Table A.2, for $M = 1.82$,

$$\frac{p_3}{p_2} = 3.698, \quad \frac{T_3}{T_2} = 1.547, \quad M_{n_3} = 0.6121$$

$$M_3 = \frac{M_{n_3}}{\sin(\beta_2 - \theta_1 - \theta_2)} = \frac{0.6121}{\sin(52.5 - 24)} = \boxed{1.28}$$

$$p_3 = \frac{p_3}{p_2}\frac{p_2}{p_1}p_1 = (3.698)(2.673)(1) = \boxed{9.88 \text{ atm}}$$

$$T_3 = \frac{T_3}{T_2}\frac{T_2}{T_1}T_1 = (1.547)(1.361)(300) = \boxed{631.6 \text{ K}}$$

The process on the pressure deflection diagram is shown below in Figure7 where the pressure change from p_1 to p_2 an finally to p_3 at point 3.

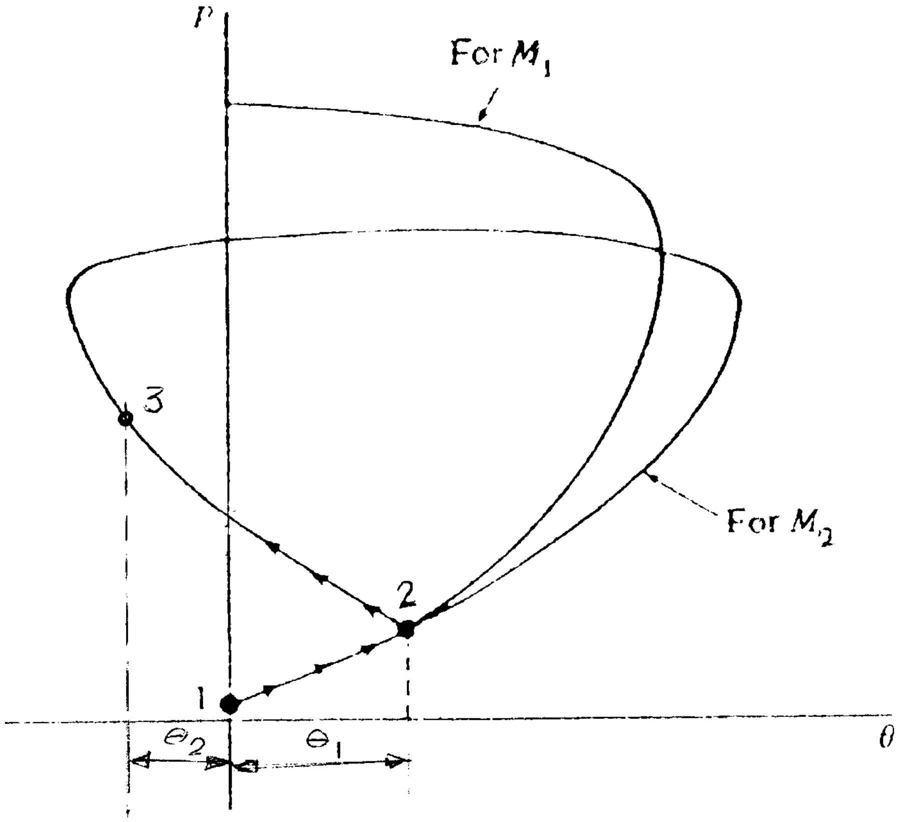

Figure 7 (reflection on pressure deflection diagram)

From $\theta-\beta-M$ **curves, for** M_3 **= 1.28** θ_{max} **= 6.5 degrees.**