Benha University
Benha Faculty of Engineering
Electrical Engineering and Circuit Analysis (b) (E1102)
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June 2013
Electrical Department
1st Year Electrical
Time: 3 Hrs

Model Answer

Question (1): [16 Marks]

The variable resistor R_o in the circuit shown in Fig (1) is adjusted until maximum average power is delivered to R_o.

a) What is the value of R_o in ohms?
b) Calculate the average power delivered to R_o.
c) If R_o is replaced with variable impedance Z_o what is the maximum average power that can be delivered to Z_o?

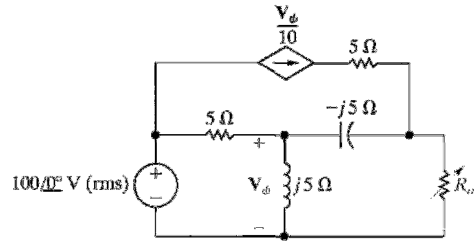
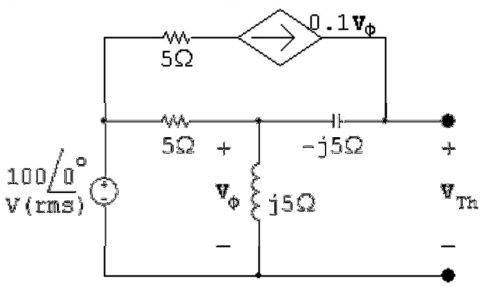


Fig (1)

[a] Open circuit voltage:

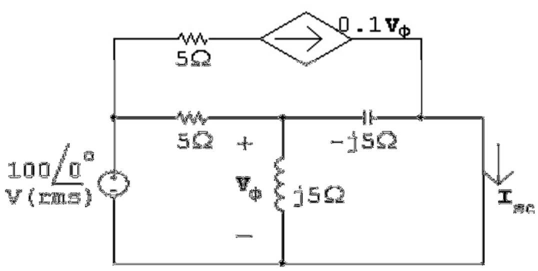


$$\frac{\mathbf{V}_\phi - 100}{5} + \frac{\mathbf{V}_\phi}{j5} - 0.1\mathbf{V}_\phi = 0$$

$$\therefore \quad \mathbf{V}_\phi = 40 + j80\,\mathrm{V(rms)}$$

$$\mathbf{V}_{\mathrm{Th}} = \mathbf{V}_\phi + 0.1\mathbf{V}_\phi(-j5) = \mathbf{V}_\phi(1 - j0.5) = 80 + j60\,\mathrm{V(rms)}$$

Short circuit current:



$$\mathbf{I}_{\mathrm{sc}} = 0.1\mathbf{V}_\phi + \frac{\mathbf{V}_\phi}{-j5} = (0.1 + j0.2)\mathbf{V}_\phi$$

$$\frac{\mathbf{V}_\phi - 100}{5} + \frac{\mathbf{V}_\phi}{j5} + \frac{\mathbf{V}_\phi}{-j5} = 0$$

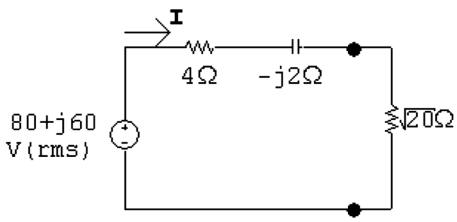
$$\therefore \quad \mathbf{V}_\phi = 100\,\mathrm{V(rms)}$$

$$\mathbf{I}_{\mathrm{sc}} = (0.1 + j0.2)(100) = 10 + j20\,\mathrm{A(rms)}$$

$$Z_{\mathrm{Th}} = \frac{\mathbf{V}_{\mathrm{Th}}}{\mathbf{I}_{\mathrm{sc}}} = \frac{80 + j60}{10 + j20} = 4 - j2\,\Omega$$

$$\therefore \quad R_o = |Z_{\mathrm{Th}}| = 4.47\,\Omega$$

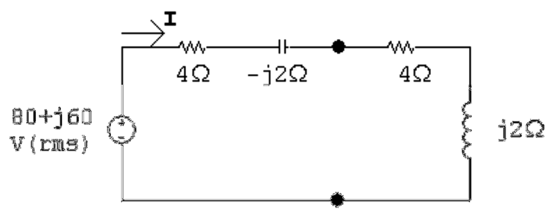
[b]



$$\mathbf{I} = \frac{80 + j60}{4 + \sqrt{20} - j2} = 7.36 + j8.82\,\text{A (rms)}$$

$$P = (11.49)^2(\sqrt{20}) = 590.17\,\text{W}$$

[c]



$$\mathbf{I} = \frac{80 + j60}{8} = 10 + j7.5\,\text{A (rms)}$$

$$P = (10^2 + 7.5^2)(4) = 625\,\text{W}$$

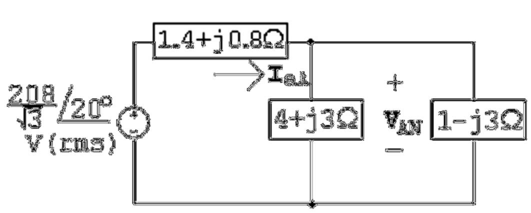
<u>Question (2):</u> [16 Marks]

In a balanced three-phase system, the source is a balanced **Y** with an ***abc*** phase sequence and a line voltage $V_{ab} = 208\,\underline{/50^\circ}$ V. The load is a balanced **Y** in parallel with a balanced Δ. The phase impedance of the **Y** is ***4 + j 3 Ω / φ*** and the phase impedance of the Δ is ***3 – j 9 Ω / φ***. The line impedance is ***1.4 + j 0.8 Ω / φ***. Draw the single phase equivalent circuit and use it to calculate the line voltage at the load in the a-phase.

$$\mathbf{V}_{\text{an}} = 1/\sqrt{3}\underline{/-30^\circ}\mathbf{V}_{\text{ab}} = \frac{208}{\sqrt{3}}\underline{/20^\circ}\text{ V (rms)}$$

$$Z_y = Z_\Delta/3 = 1 - j3\,\Omega$$

The a-phase circuit is



$$Z_{\text{eq}} = (4 + j3)\|(1 - j3) = 2.6 - j1.8\,\Omega$$

$$\mathbf{V}_{\text{AN}} = \frac{2.6 - j1.8}{(1.4 + j0.8) + (2.6 - j1.8)}\left(\frac{208}{\sqrt{3}}\right)\underline{/20^\circ} = 92.1\underline{/-0.66^\circ}\text{ V (rms)}$$

$$\mathbf{V}_{\text{AB}} = \sqrt{3}\underline{/30^\circ}\mathbf{V}_{\text{AN}} = 159.5\underline{/29.34^\circ}\text{ V (rms)}$$

<u>Question (3):</u> [14 Marks]

A 100 V ABC system is connected to the load shown in Fig (2). Find the readings of the meter,

a) If it is a high-impedance voltmeter.
b) If it is a very low impedance ammeter.

Solved exactly in lectures;

a) Replace the high impedance voltmeter with open circuit
b) Replace the very low impedance ammeter with short circuit

Write the mesh current equations to find the three line currents then find the reading of the meter.

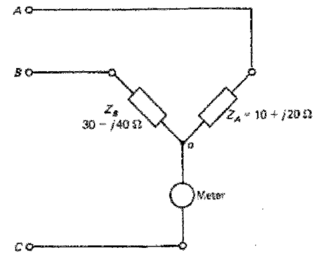


Fig (2)

<u>Question (4):</u> [12 Marks]

The resistor $\boldsymbol{R_f}$ in the circuit in Fig (3) is adjusted until the ideal op amp saturates. Specify $\boldsymbol{R_f}$ in kΩ.

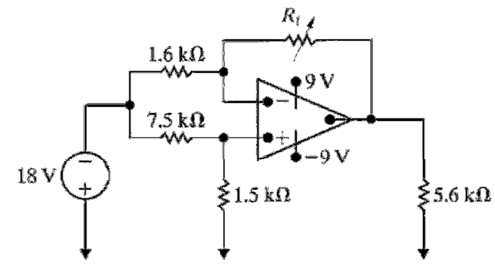


Fig (3)

The voltage at the positive terminal of the op-amp v_p and the voltage at the negative terminal is v_n

$$v_p = \frac{1500}{9000}(-18) = -3 \text{ V} = v_n$$

$$\frac{-3+18}{1600} + \frac{-3-v_o}{R_f} = 0$$

$$\therefore \quad v_o = 0.009375 R_f - 3$$

$$v_o = 9 \text{ V}; \qquad R_f = 1280\,\Omega$$

$$v_o = -9 \text{ V}; \qquad R_f = -640\,\Omega$$

$$\text{But} \quad R_f \geq 0, \qquad \therefore \quad R_f = 1.28\,\text{k}\Omega$$

<u>Question (5):</u> [10 Marks]

Assume that the initial energy stored in the inductors of Fig (4) is zero. Find the equivalent inductance with respect to the terminals ***a, b***.

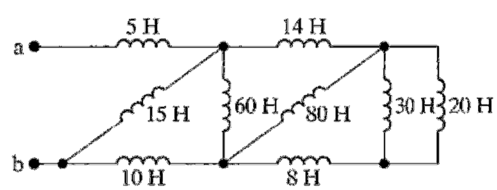


Fig (4)

$$30 \| 20 = 12\,\text{H}$$

$$80 \| (8+12) = 16\,\text{H}$$

$$60 \| (14+16) = 20\,\text{H}$$

$$15 \| (20+10) = 20\,\text{H}$$

$$L_{ab} = 5 + 10 = 15\,\text{H}$$

Question (6): [12 Marks]

Both switches in the circuit in Fig (5) have been closed for a long time. At $t = 0$, both switches open simultaneously.

a) Find $i_o(t)$ for $t \geq 0^+$.

b) Find $v_o(t)$ for $t \geq 0$.

c) Calculate the energy (in micro-joules) trapped in the circuit.

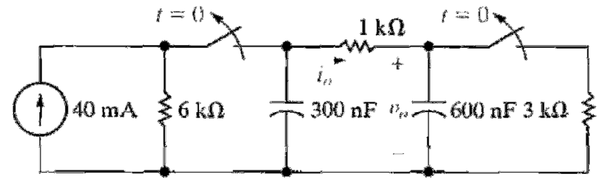
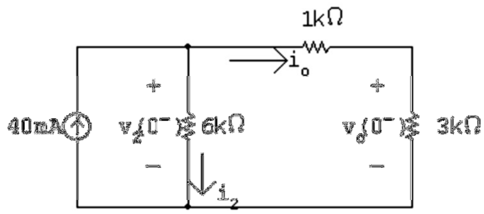


Fig (5)

The capacitors are open circuit for t < 0,

[a] $t < 0$:



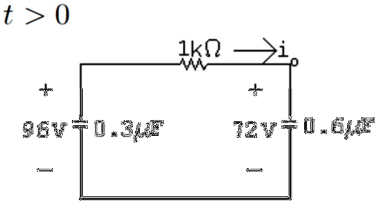
$$i_o(0^-) = \frac{6000}{6000 + 4000}(40\,\text{m}) = 24\,\text{mA}$$

$$v_o(0^-) = (3000)(24\,\text{m}) = 72\,\text{V}$$

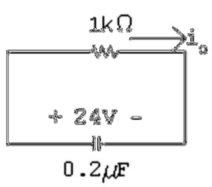
$$i_2(0^-) = 40 - 24 = 16\,\text{mA}$$

$$v_2(0^-) = (6000)(16\,\text{m}) = 96\,\text{V}$$

$t > 0$

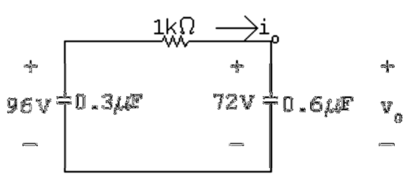


$$\tau = RC = (1000)(0.2 \times 10^{-6}) = 200\,\mu\text{s}; \qquad \frac{1}{\tau} = 5000$$



$$i_o(t) = \frac{24}{1 \times 10^3} e^{-t/\tau} = 24e^{-5000t}\,\text{mA}, \qquad t \geq 0^+$$

[b]



$$v_o = \frac{1}{0.6 \times 10^{-6}} \int_0^t 24 \times 10^{-3} e^{-5000x}\, dx + 72$$

$$= (40{,}000)\frac{e^{-5000x}}{-5000}\Bigg|_0^t + 72$$

$$= -8e^{-5000t} + 8 + 72$$

$$v_o = [-8e^{-5000t} + 80]\,\text{V}, \qquad t \geq 0$$

[c] $w_{\text{trapped}} = (1/2)(0.3 \times 10^{-6})(80)^2 + (1/2)(0.6 \times 10^{-6})(80)^2$

$w_{\text{trapped}} = 2880\,\mu\text{J}.$

Check:

$$w_{\text{diss}} = \frac{1}{2}(0.2 \times 10^{-6})(24)^2 = 57.6\,\mu\text{J}$$

$$w(0) = \frac{1}{2}(0.3 \times 10^{-6})(96)^2 + \frac{1}{2}(0.6 \times 10^{-6})(72)^2 = 2937.6\,\mu\text{J}.$$

$$w_{\text{trapped}} + w_{\text{diss}} = w(0)$$

$$2880 + 57.6 = 2937.6 \qquad \text{OK.}$$

Question (7): [10 Marks]

The source ***v(t) = 1.414 cos(ωt) V*** is applied to a three-branch ***RLC*** parallel circuit, where ***R = 100 Ω***, ***L = 0.2 mH***, and ***C = 0.22 µF***, find:

a) The resonance frequency, ***Q*** and the bandwidth of this circuit.
b) The branch currents and the source current at the resonance frequency in the time domain. Express these currents in phasor form and draw the phasor diagram.

given R, L, and C values then,

[a] $\omega_o = \frac{1}{\sqrt{LC}} = 150.756\ Krads$

Quality Factor, $Q = \frac{R}{2\pi f L} = 2\pi f CR = R\sqrt{\frac{C}{L}}$

:. Q = 3.316

BW = ω_o / Q = 45.455 Krads

[b] We remember that the total current flowing in a parallel RLC circuit is equal to the vector sum of the individual branch currents and for a given frequency is calculated as:

$$I_R = \frac{V}{R}$$

$$I_L = \frac{V}{X_L} = \frac{V}{2\pi f L}$$

$$I_C = \frac{V}{X_C} = V.2\pi f C$$

$$\text{Therefore,}\quad I_T = \text{vector sum of } (I_R + I_L + I_C)$$

$$I_T = \sqrt{I_R^2 + (I_L + I_C)}$$

At resonance, currents I_L and I_C are equal and cancelling giving a net reactive current equal to zero. Then at resonance the above equation becomes.

$$I_T = \sqrt{I_R^2 + 0^2} = I_R$$

From the equations; calculate the currents and draw the Phasor diagram.

With best wishes