المادة : التحكم الآلى   م ٤٨٢ نموذج الاجابة والاسئلة

التاريخ الاثنين ١٦ يناير ٢٠١٧

أستاذ المادة : د. محمد عبد اللطيف الشرنوبى

Benha University
College of Engineering at Banha
Mechanical Eng. Dept.
Subject :Automatic Control
Questions For Final Examination

3rd Year Mechanics
January/16/2017

Examiner : Dr. Mohamed Elsharnoby **Time :180 min.**

Attempt all questions, Number of questions = 4, Number of pages = 2

1-aDerive the transfer function of the system described by the following block diagram;

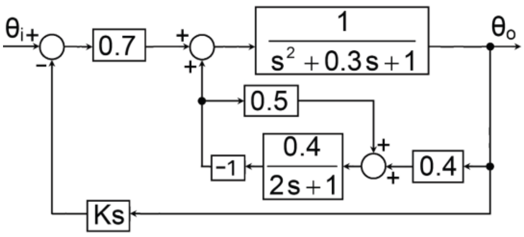


Figure 1

b) Draw the signal flow graph of the system represented by the given block diagram and find its transfer function by using Masson rule rule ti find the transfer function of the system whose SFG is given in figure 2.

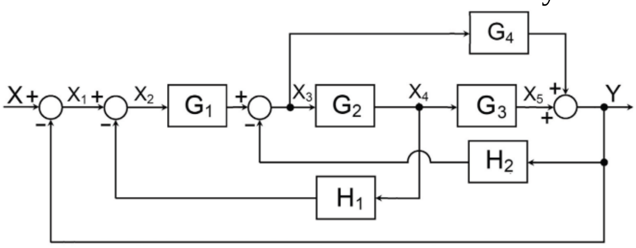


Figure 2

c) If the transfer function of a system is given by:

$$\frac{Y(s)}{U(s)} = \frac{S^2 + 2S + 1}{S^4 + 2S^3 + 5S^2 + 3S + 7}$$

i) Draw a signal flow graph represents this system.
ii) Deduce the state space representation of the system.
iii) Write the differential equation of the system in canonical form.

2-a) The following are two block diagrams of two control systems (Figure 3). Find the natural frequency, damping ratio, damped natural frequency, settling time and maximum percentage overshoot for each of them.

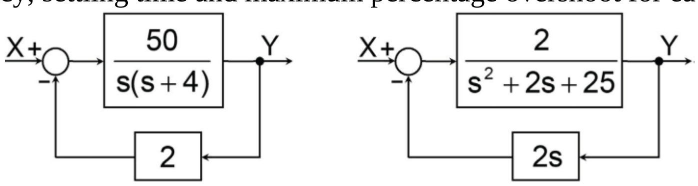


Figure 3

b) Find the value of constant K for the critical damping of the system described by the following block diagram.

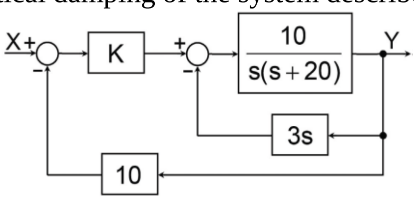


Figure 4

c) Consider the standard feedback system shown in Figure 5.

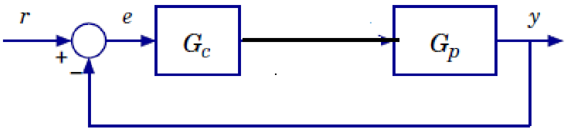


Figure 5

Let the plant transfer function is given as

$$G_p(s) = \frac{1}{s(s+5)}.$$

Design a Proportional plus Integral (PI) controller of the form

$$G_c(s) = K_P + \frac{K_I}{s}$$

such that the closed loop system is stable with one pole at $r_1 = -4$ and the other two poles are at $r_{2,3} = -\zeta\omega_o \pm j\omega_o\sqrt{1-\zeta^2}$ with $\omega_o \geq 2$, $\zeta > 0$, and ζ is as large as possible.

3-a)

Plot the Nyquist and Bode plots of the open loop of the system shown in Figure 6 with and without compensation and discuss the effect of introducing a phase lead compensator having the following transfer function:

$$G_c = \frac{0.5s+1}{0.02s+1}$$

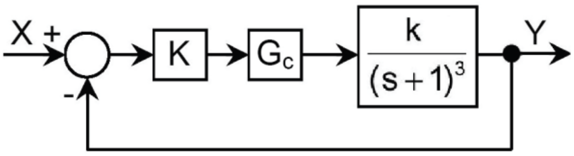


Figure 6

b) For the system described by the block diagram shown in figure 7

(i) Find the values of constant k for a steady state error ess = 0.1 and for ess = 0.05.

(ii) Derive the expressions for the open loop gain and phase.

(iii) Calculate the phase cross-over frequency and find the value of constant k if the gain margin is 10 dB

(iv) Calculate the gain cross-over frequency and phase margin for the value of k calculated in (iii)

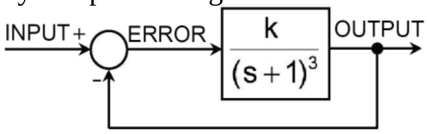


Figure 7

4-a) Plot the root locus of the following system in Figure 8 and sketch the response of the closed loop to unit step input for K=1 & K=100.

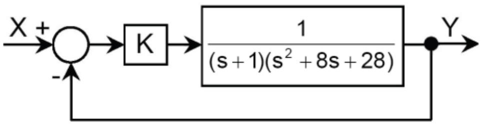


Figure 8

b) Figure 9 shows the asymptotic gain Bode plot of a third order system having a transfer function of the form:

$$G(s) = \frac{k}{s(1+T_1 s)(1+T_2 s)}$$

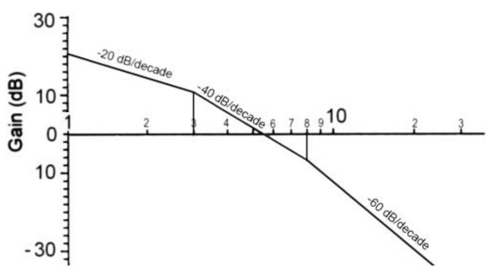


Figure 8

(i) Find the numerical values of the constants k, T_1 and T_2.
(ii) Plot the phase diagram.
(iii) Find graphically the gain and phase margins.
(iv) Find analytically the gain margin.
(v) What is the type and order of the system?

GOOD LUCK

Benha University

Mechanical Eng. Dept. **4^{th} Year Mechanics**
Subject :Automatic Control **Date 16/1/2017**

Model Answer of The Final Examination

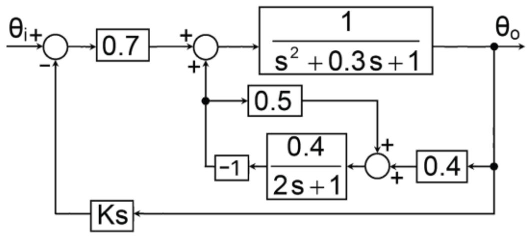
Elaborated by: Dr. Mohamed Elsharnoby

المادة : التحكم الآلى م ٤٨٢ نموذج الاجابة

التاريخ الاثنين ١٦ يناير ٢٠١٧

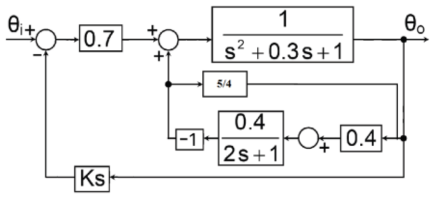
أستاذ المادة : د. محمد عبد اللطيف الشرنوبى

1-a)

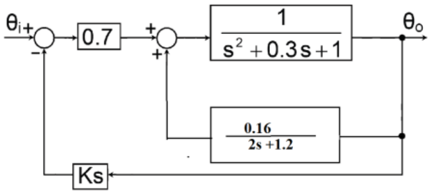


Using the block diagram reduction

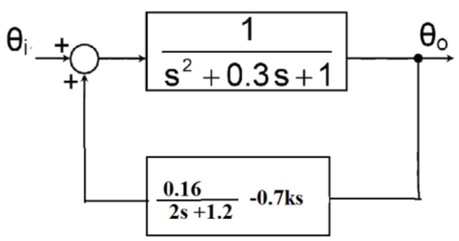
(a)



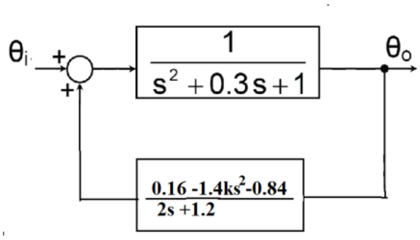
(b)



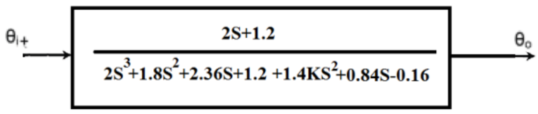
(c)



(d)



(e)

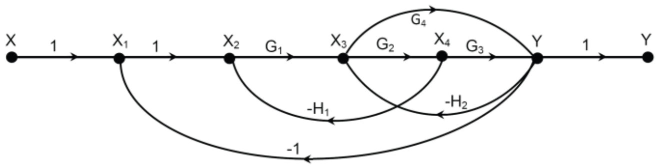


The transfer function is

$$\frac{2S+1.2}{2S^{3}+(1.4K+1.8)S^{2}+3.2S+1.04}$$

1-b

1. Signal Flow Graph



2. Loops and paths gains

$M_1 = G_1G_2G_3$ $\qquad$ $M_2 = G_1G_4$

$\varphi_1 = -G_1G_2G_3$ $\qquad$ $\varphi_2 = -H_1G_1G_2$

$\varphi_3 = -H_2G_2G_3$ $\qquad$ $\varphi_4 = -H_2G_4$

$\varphi_5 = -G_1G_4$

3. Application of Mason's formula

$$\Delta = 1 - \varphi_1 - \varphi_2 - \varphi_3 - \varphi_4 - \varphi_5 \qquad \Delta_1 = 1 \qquad \Delta_2 = 1$$

$$G(s) = \frac{G_1G_2G_3 + G_1G_4}{1 + G_1G_2G_3 + H_1G_1G_2 + H_2G_2G_3 + H_2G_4 + G_1G_4}$$

1-b)

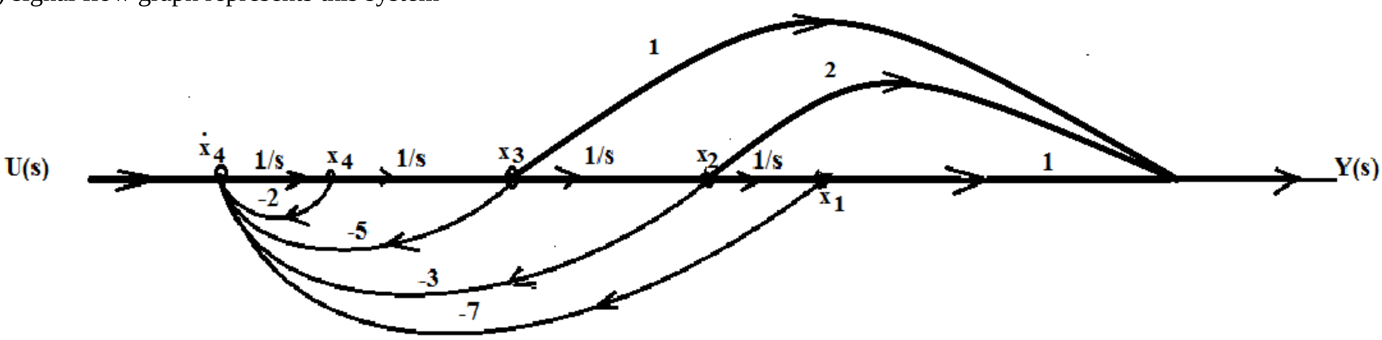
If the transfer function of a system is given by:

$$\frac{Y(s)}{U(s)} = \frac{S^{2} + 2S + 1}{S^{4} + 2S^{3} + 5S^{2} + 3S + 7}$$

Devide both numenator and denomenator by $\mathbf{s^6}$

we get $$TF = \frac{\frac{1}{s^2} + \frac{2}{s^3} + \frac{1}{s^4}}{1 + \frac{2}{s} + \frac{5}{s^2} + \frac{3}{s^3} + \frac{7}{s^4}}$$

i) signal flow graph represents this system

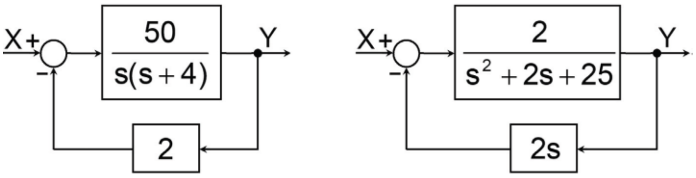


Ii)Deduce the state space representation of the system., iii

$$\dot{X} = AX + BU = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -2 & -5 & -3 & -7 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} U$$

$$Y = CX + DU = \begin{bmatrix} 1 & 2 & 1 \end{bmatrix} X + O.U$$

2-a)



(a) (b)

The transfer function for the system at (a) is

$$\frac{50}{s^2 + 4s + 100}$$

$\omega_n^2 = 100$ and $\zeta = 0.2$

The damped frequency ω_d is given by :

$$\omega_d = \omega_n \sqrt{1 - \zeta^2}$$

Settling time is the time required for the response to reach an end state within 5% of final, steady state, value. For second order system, the settling time is:

$$\boxed{t_s \le \frac{3 - 0.5\ln(1 - \zeta^2)}{\zeta\omega_n}} \quad \text{Or} \quad \boxed{t_s \cong \frac{3}{\zeta\omega_n}}$$

Maximum overshoot is usually expressed in percentage of the steady state value of the output.

$$\text{MPO} = 100e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}}\ \%$$

The transfer function for the sytem at (b) is

$$\frac{2}{s^2 + 6s + 25}$$

$\omega_n^2 = 25$ and $\zeta = 0.6$

Settling time is the time required for the response to reach an end state within 5% of final, steady state, value. For second order system, the settling time is:

$$\boxed{t_s \le \frac{3 - 0.5\ln(1 - \zeta^2)}{\zeta\omega_n}} \quad \text{Or} \quad \boxed{t_s \cong \frac{3}{\zeta\omega_n}}$$

Maximum overshoot is usually expressed in percentage of the steady state value of the output.

$$\text{MPO} = 100e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}}\,\%$$

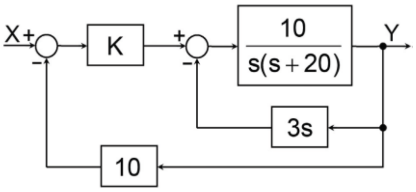
For system at (a) $\omega_n^2 = 100$ and $2\zeta\,\omega_n = 4 \longrightarrow \omega_n=10$, $\zeta=0.2$, $\omega_d=9.7979$, $t_s=1.5$ sec, and MPO =52.66%

For system at (b) $\omega_n^2 = 25$ and $2\zeta\,\omega_n = 6 \longrightarrow \omega_n=5$, $\zeta=0.6$, $\omega_d=4$, $t_s=1$ sec, and MPO =9.47% **Settling time** is the time required for the response to reach an end state within 2% of final, steady state, value. For second order system, the settling time is:

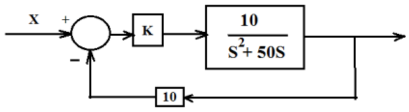
$$\boxed{t_s \cong \frac{4}{\zeta\omega_n}}$$

For system (a) $t_s = 2$ sec
For system (b) $t_s = 1.33$ sec

2-b) Find the value of constant K for the critical damping of the system described by the following block diagram.



The block diagram is reduced to

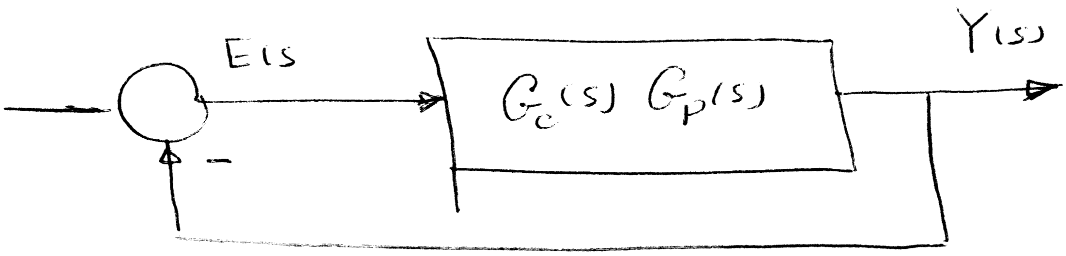


The transfer function is

$$\frac{10\,K}{S^2 + 50S + 100K}$$

For critical damping $\zeta = 1 \longrightarrow \; 2\,\omega_n = 50, \;\; \omega_n^2 = 100k \longrightarrow k=6.25$

2-c) Consider the standard feedback system shown below in figure



The characteristic Eqn. is given by:

$$1 + G_c(s)\,G_p(s) = 0$$

$$1 + \frac{k_p s + k_i}{s}\,\frac{1}{(s^2+5s)} = 0$$

$$s^3 + 5s^2 + k_p s + k_i = 0$$

$s = -4$ satisfies The eqn.

$$-64 + 80 - 4k_p + k_i = 0$$

$$16 - 4k_p + k_i = 0$$

$$k_i = 4k_p - 16$$

$$s^3 + 5s^2 + k_p s + 4k_p - 16 = 0$$

$$k_p > 4$$

$$(s^3 + 4s^2) + (s^2 - 16) + k_p(s+4) = 0$$

$$s^2(s+4) + (s+4)(s-4) + k_p(s+4) = 0$$

$$(s+4)\left[s^2 + s + (k_p - 4)\right] = 0$$

$$r_{2,3} = \frac{-1 \pm \sqrt{1 + 16 - 4k_p}}{2}$$

$$-\frac{1}{2} \pm j\sqrt{k_p - \frac{17}{4}} = -\xi\omega_n \pm j\,\omega_n\sqrt{1-\xi^2}$$

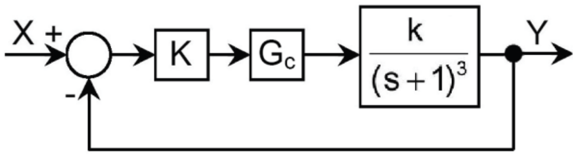
choose $\omega_n = 2 \longrightarrow \xi = 0,25$

$$4(1-\xi^2) = k_p - \frac{17}{4} \Rightarrow \frac{15}{4} + \frac{17}{4} = k_p \Rightarrow$$

$$\boxed{k_p = 8}$$

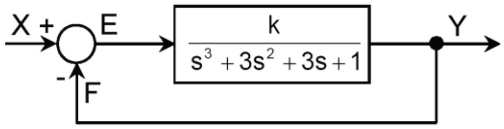
3-a) Plot the Nyquist and Bode plots of the open loop of the system shown in Figure 6 with and without compensation and discuss the effect of introducing a phase lead compensator having the following transfer function:

$$G_c = \frac{0.5s+1}{0.02s+1}$$



solution

Apply Routh-Hurwitz stability criterion to find the limiting value of the open loop gain, for system stability and discuss the obtained results.



The closed loop characteristic equation is:

$$s^3+3s^2+3s+1+k=0$$

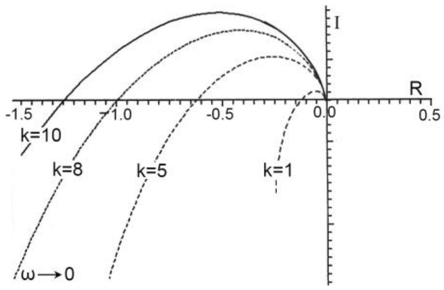
$$\begin{array}{c|ll} 3 & 1 & 3 \\ 2 & 3 & 1+k \\ 1 & 3-\dfrac{1+k}{3} & 0 \\ 0 & 1+k & \end{array}$$

The system is stable if $3-\dfrac{1+k}{3}>0$ and $1+k>0$

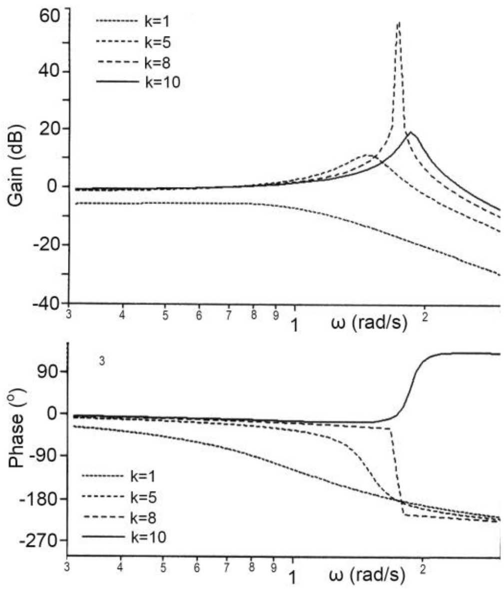
Or $8 > k > -1$

Nyquist plot of the open loop of transfer function for different values of gain;

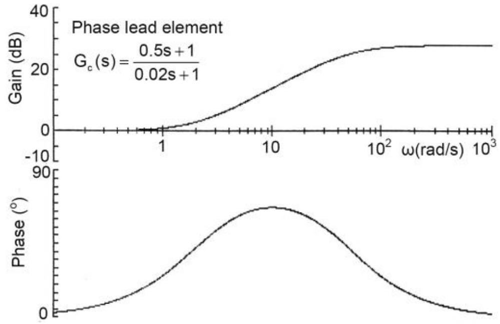
$$GH(s)=\frac{k}{s^3+3s^2+3s+1}$$



The open loop Bode plot



Adding the lead compensator given by the equation $G_c=\dfrac{0.5s+1}{0.02s+1}$ will add the compensator Bode plot below to the above Bode plot.

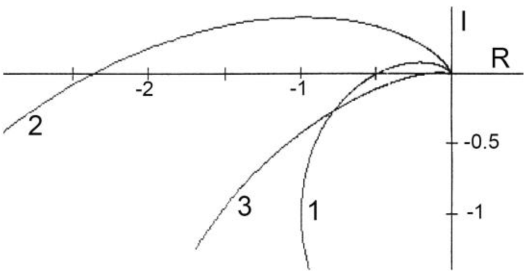


For a phase lead element having T =0.02 sec and a =25, the frequency response parameters are calculated as follows.

$$a = 25, \quad T = 0.02\ \text{sec}.$$

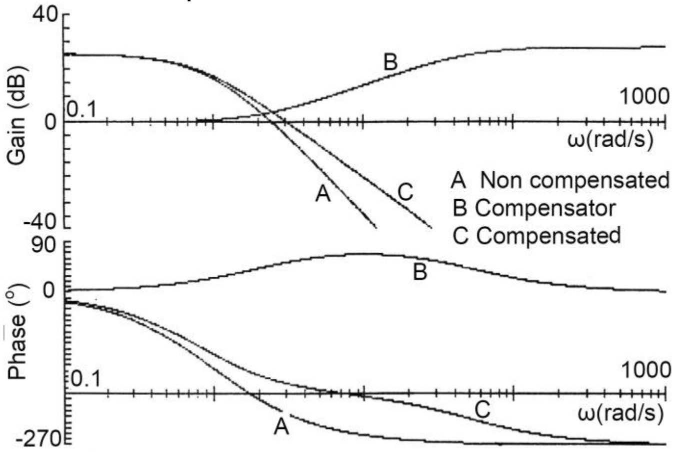
$$\text{Corner frequencies} \quad \omega_1 = \frac{1}{aT} = 2\,(1/\text{sec}) \quad \text{and} \quad \omega_2 = \frac{1}{T} = 50\ (1/\text{sec})$$

$$\omega_m = 10\,(1/\text{sec}), \quad \varphi_{max} = \sin^{-1}\frac{25-1}{25+1} = 67.38^\circ \quad \text{and} \quad \varphi\left(\frac{1}{T}\right) = \varphi\left(\frac{1}{aT}\right) = 42.7^\circ$$



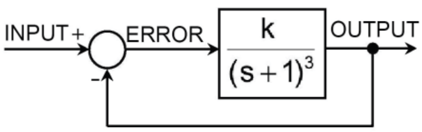
(1) K=4, Stable with GM=6 dB and PM=27° (2) K=19, Unstable without compensator
(3) K=19, Stable with phase lead compensator, GM=15 dB and PM=22°

Nyquist plot of the non-compensated open loop system and of the system with series connected phase lead compensator.



Bode plot of the non-compensated open loop system and system with series connected phase lead compensator.

3-b)



(i)

$$k_p = \lim_{s\to 0} G(s)H(s) = \lim_{s\to 0} \frac{k}{(s+1)^3} = k$$

$$k_p = \frac{1-\bar{e}_{ss}}{\bar{e}_{ss}}$$

For $\bar{e}_{ss} = 0.1$, $k_p = 9 = k$ or $k = 9$

For $\bar{e}_{ss} = 0.05$, $k_p = 19 = k$ or $k = 19$

The increase in gain reduces the steady state error. For $\bar{e}_{ss} = 0.05$, the open loop gain k should be: $k = 19$

(ii)

$$\text{Gain} = |G(i\omega)| = \frac{k}{\left(\sqrt{1+\omega^2}\right)^3}$$

$$\text{Phase} = \angle G(i\omega) = -3\tan^{-1}(\omega)$$

(iii)

At the phase cross-over frequency, $\omega = \omega_{cp}$ the phase φ=-180°

$$-180^\circ = -3\tan^{-1}(\omega_{cp}) \qquad \text{Or} \qquad \omega_{cp} = \sqrt{3}\ \text{rad/s}$$

$$\text{Gain} = g = \frac{k}{\left(\sqrt{1+\omega_{cp}^2}\right)^3} = \frac{k}{8}$$

For a gain margin of 10 dB; GM=10=20 Log (1/g)

Or $g = 1/\sqrt{10}$, Then $k = 8/\sqrt{10} = 2.53$

For gain margin GM=10 dB, the open loop gain should be k=2.53

(iv)

At the gain cross-over frequency, $\omega = \omega_{cg}$, g=1

$$g = |G(i\omega_{cg})| = \frac{k}{\left(\sqrt{1+\omega_{cg}^2}\right)^3} = 1$$

For k=2.53, $\omega_{cg} = 0.926\ \text{rad/s}$

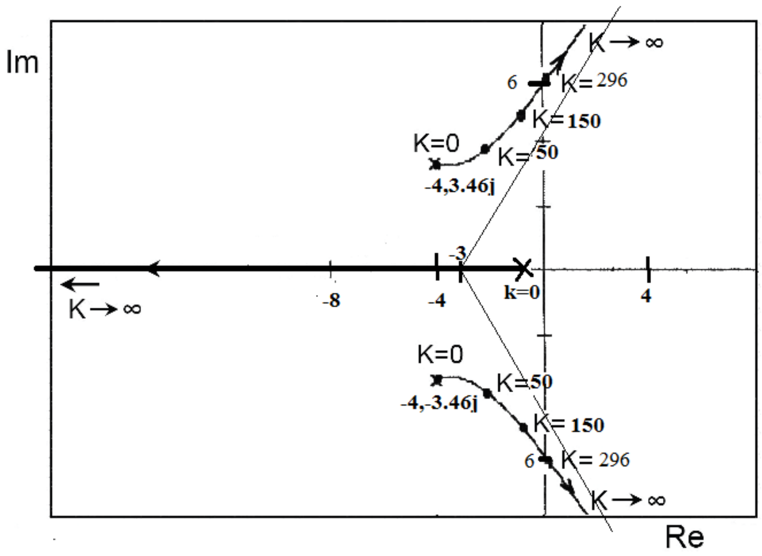
At this frequency, the phase φ is calculated as follows.

$$\varphi = -3\tan^{-1}(\omega_{cg}) = -128.36^\circ$$

The phase margin PM=180°+ $\angle G(i\omega_{cg})$

Or $PM = 180^\circ - 128.36^\circ = 51.64^\circ$

4-a)



The asymptotes intesect the real line at -3
There is no break points
The intervals on the real line that lie on the root locus are (-1, -∞),
Find K that the system start to be unstable where the root locus intersect the imaginary axis
The characteristic equation is

$$S^3+9S^2+36S+K+28 = 0$$

Construct routh array

$$\begin{array}{|ll} 1 & 36 \\ 9 & k+28 \\ \frac{296-k}{9} & 0 \\ k+28 & \end{array}$$

The system is stable for k<296
For k +ve, the damped frequency is

$$\omega_d > \sqrt{12}$$

For k=296
The characteristic equation is given as $S^3+9S^2+36S+324=0$
The eouation is factorized as $(S+9)(S^2+36)=0$ and the root locus intersects the imaginary axis at ±6

For K=1, the characteristic equation is

$$S^3+9S^2+36S+29 = 0$$

And the transfer function is

$$\frac{1}{S^3+9S^2+36S+29}$$

For K=100, the characteristic equation is

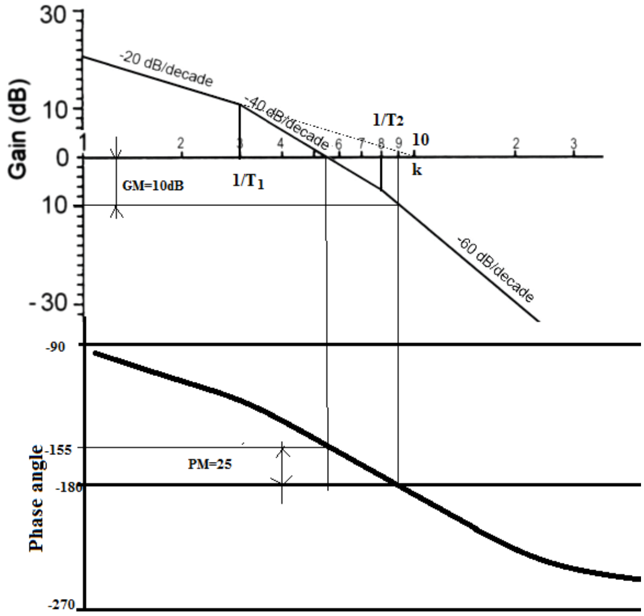
$$S^3+9S^2+36S+128=0$$

And the transfer function is

$$\frac{100}{S^3+9S^2+36S+128}$$

For k=1 the final value will reach 1/29
For k=100 the final value will reach 100/128

b)



(i) Find the numerical values of the constants k, T_1 and T_2.
(ii) Plot the phase diagram.
(iii) Find graphically the gain and phase margins.
(iv) Find analytically the gain margin.
(v) What is the type and order of the system?

i) K is the value of ω at the intersection of the first line which is 10, $T_1 = 1/3$ and $T_2=1/8$
ii) Phase diagram will start at -90 degree and end at -270
iii) GM=-10db =
iv) The gain margin you put the imaginary part of G(iω)=0i.e IM G(iω)=0
v) The system is type one.

The transfer function is

$$\frac{10}{S(\frac{s}{3}+1)(\frac{s}{8}+1)}$$

The denominator will be $S^3/24+11S^2/24+S$
The imaginary part is $-\omega^3/24+\omega$

vi) The Im G(iω)=0 if $\omega^2=24$ ad hence $|G(i\omega)| = 10/11$
The GM= 1.1