نموذج الاجابة والاسئلة مادة : التحكم الآلى م 482

التاريخ الثلاثاء 21 مايو 2019

أستاذ المادة : د. محمد عبد اللطيف الشرنوبى

الفرقة الرابعة ميكانيكا قوى

Benha University

College of Engineering at Banha

Mechanical Eng. Dept. **4th Year Mechanics**

Subject :Automatic Control (M 482) **May/21/2019**

Questions For Final Corrective Examination

Examiner : Dr. Mohamed Elsharnoby **Time :180 min.**

Attempt all questions, Number of questions = 4, Number of pages = 2

1-a) 1-a) For the system whose signal flow graph is shown in Figure 1, Find:

(i) the transfer function

(ii) The steady state error for unit input

(iii) The expression of y(t) for unit input

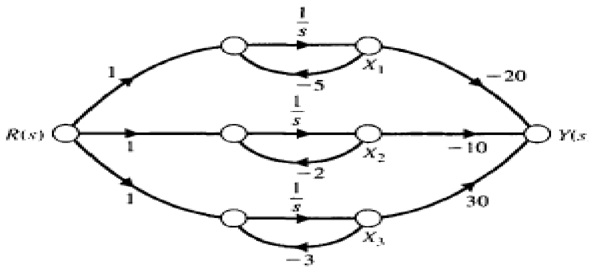

Figure 1

1-b) If the transfer function of a system is given by:

$$\frac{Y(s)}{U(s)} = \frac{S^2 + S + 2}{S^3 + 4S^2 + 5S + 2}$$

i) Obtain a state-space equation and output equation for the system defined by

ii) Draw a signal flow graph represents this system.

2- a) Determine whether the standard feedback system is stable for the following cases, and justify your answers. If the feedback system is unstable determine how many of its poles are in the right half plane.

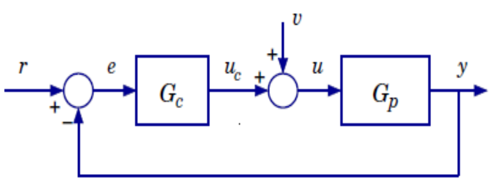

Figure 3

(a) $G_c(s) = \frac{10}{(s+1)}$; $G_p(s) = \frac{2}{(s+1)(s+2)}$

(b) $G_c(s) = \frac{5\,(s+1)}{s}$; $G_p(s) = \frac{1}{s-2}$

(c) $G_c(s) = \frac{5\,(0.1s+1)}{s}$; $G_p(s) = \frac{(s-2)^2}{(s^2+6s+10)(s+5)}$

b) What is the type of the system for the given three casees, determine the steady state error for unit step, ramp, and acceleration inputs.

3-a) Consider a unity gain feedback control system. The plant transfer function is G(s)=1/(s^2+5s+6). Let the controller be of the form C(s) =K(s+z)/(s+p). Design the controller (ie choose K, z, p>0) so that the closed loop system has poles at $-1 \pm j$

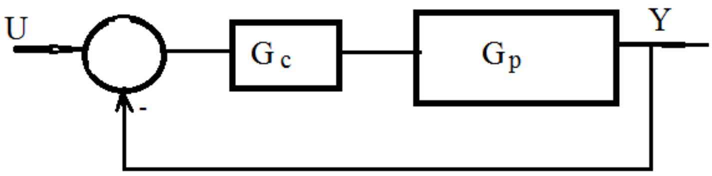

Figure 2

3-b) For the control system shown in figure 3, sketch the root locus for the following three cases, indicate its direction, where K = 0, where K = ∞, and if they exist, find asymptotes.

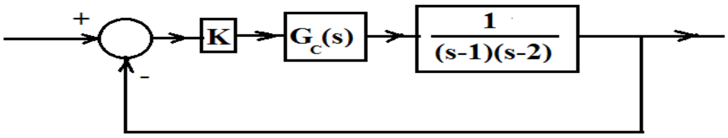

Figure 3

(i) **$G_c(s) = 1$. , (ii) $G_c(s) = s+1$ (PD compensation), (iii)$G_c(s) = 1+1/s$ (PI compensation).**

For the appropriate choice of compensator, use root locus and Routh Hurwitz techniques to find the range of K for an under damped response.

4-a) Bode Plots of a stable plant $G_P(s)$ are shown in Figure 4 below. Design a proportional controller $G_c(s) = K$, so that the steady state error for a unit step input is as small as possible, and the gain margin of the feedback system is greater or equal to 5 db.

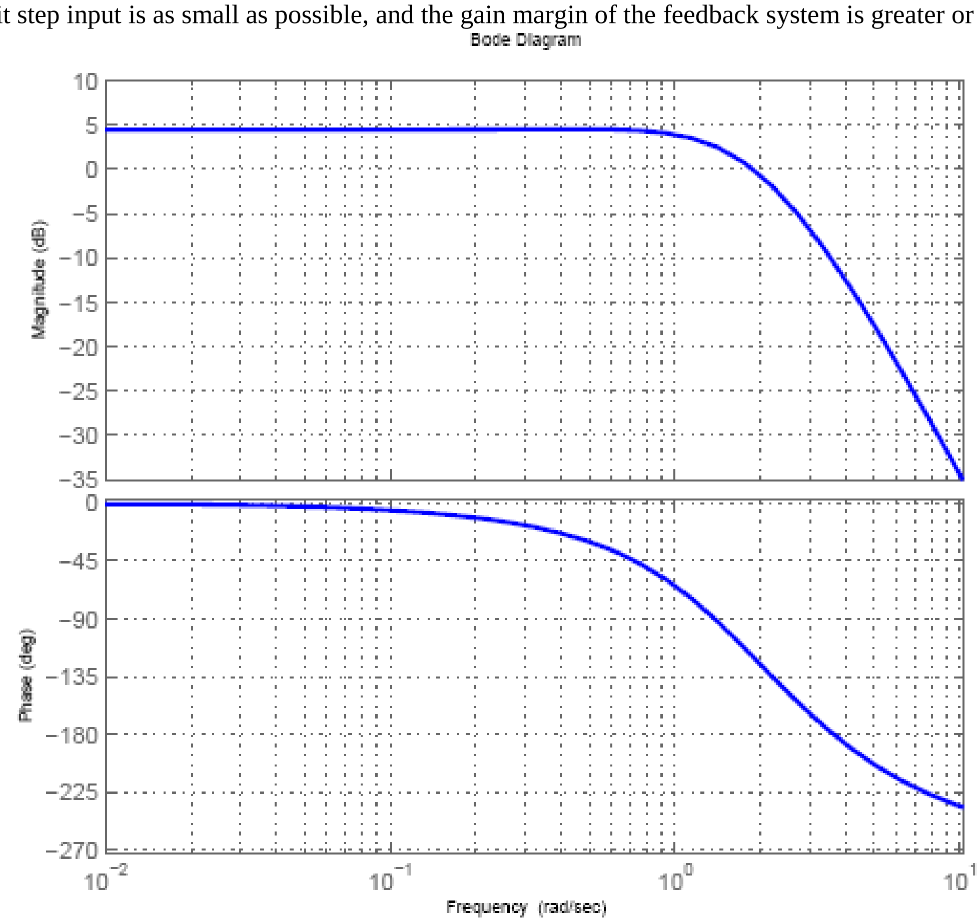

Figure 4

4-b) Consider the following system where G(s) is a transfer function. Asymptotic Bode plots of G(s) is given in figure 5 below. For calculations, you may use these Asymptotic plots.

i) Find the gain margin of the system.

ii) Find the phase margin of the system.

iii) Find the transfer function of the system. .

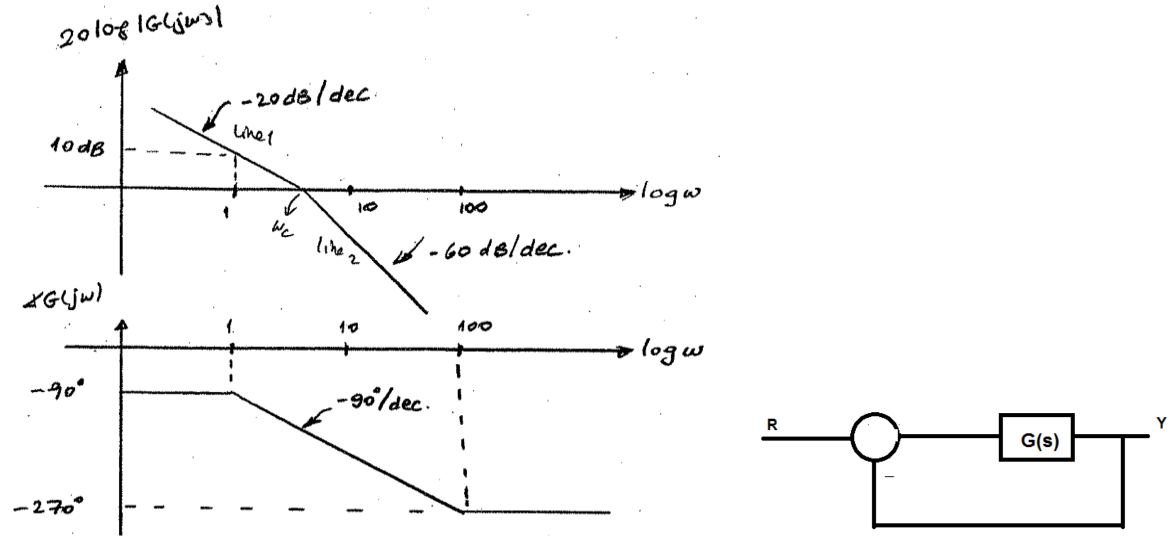

Figure 5

Note : justfy your answer

G O O D   L U C K

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Subject :Automatic Control M482 **May 21/2019**
Model Answer of the Final Corrective Examination
Elaborated by: Dr. Mohamed Elsharnoby

نموذج اجابة

المادة : التحكم الآلى م 482

أستاذ المادة : د. محمد عبد اللطيف الشرنوبى

ألفرقة الرابعة ميكانيكا نظام قديم

1-a) The signal flow graph of the sysytem shown in Figure 1. List all loops, , and use Masson rule to find the transfer function of the given system.

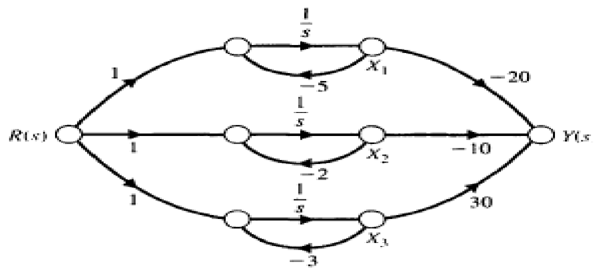


Figure 1

Loops are $L_1 = -5/s$, $L_2 = -2/s$, $L_3 = -3/s$

Paths are $M_1 = -20/s$, $M_2 = -10/s$, $M_3 = 30/s$

$\Delta = 1 - (L_1 + L_2 + L_3) + (L_1L_2 + L_1L_3 + L_2L_3) - (L_1L_2L_3)$

$\Delta = 1 + 10/s + 31/s^2 + 30/s^3$

$\Delta_1 = 1 + 5/s + 6/s^2$

$\Delta_2 = 1 + 8/s + 15/s^2$

$\Delta_3 = 1 + 7/s + 10/s^2$

$$\mathrm{TF} = \frac{M_1\Delta_1 + M_2\Delta_2 + M_3\Delta_3}{\Delta} = \frac{\frac{30}{s^2} + \frac{30}{s^3}}{1 + 10/s + 31/s^2 + 30/s^3} = \frac{30(s+1)}{s^3 + 10s^2 + 31s + 30}$$

ii) For unit step input R(s)=1/s

then $Y(s) = \left[\dfrac{30(s+1)}{s^3+10s^2+31s+30}\right]\dfrac{1}{s}$

The steady state value of Y = $y(\infty) = \lim_{s\to 0} sY(s) = 1$

The steady state error is 0

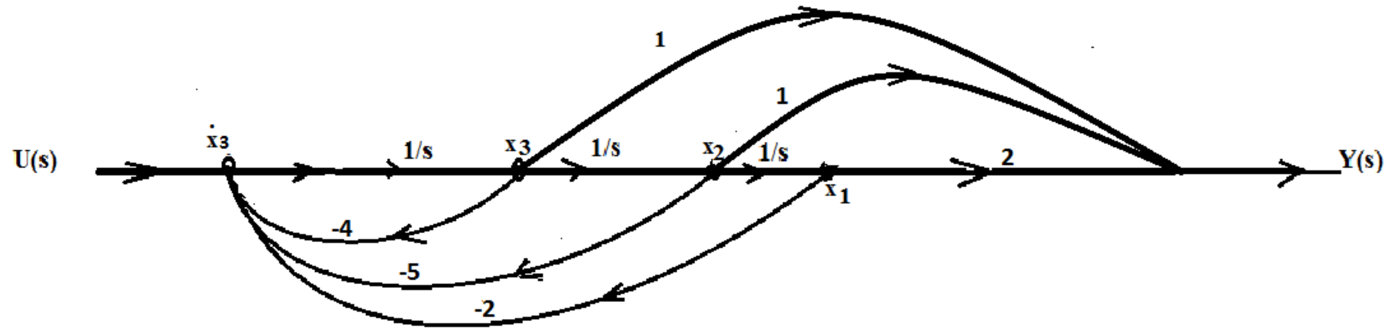
1-b)
If the transfer function of a system is given by:

$$\frac{Y(s)}{U(s)} = \frac{S^2+S+2}{S^3+4S^2+5S+2}$$

Devide both numenator and denomenator by $\mathbf{s^3}$

we get $$TF = \frac{\frac{1}{s}+\frac{1}{s^2}+\frac{2}{s^3}}{1+\frac{4}{s}+\frac{5}{s^2}+\frac{2}{s^3}}$$

i) signal flow graph represents this system



Ii)Deduce the state space representation of the system., iii

$$\mathbf{\dot{X} = AX+BU =} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -4 & -5 & -2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \mathbf{U}$$

$$\mathbf{Y = CX + DU =} \begin{bmatrix} 2 & 1 & 1 \end{bmatrix} \mathbf{X} \ \mathbf{+ O.U}$$

2-a)

In all cases we need to apply the Routh-Hurwitz test

(a) $(s+1)^2(s+2)+20 = (s^2+2s+1)(s+2)+20 = s^3+4s^2+5s+22$

s^3	1	5
s^2	4	22
* s^1	-2	0
s^0	22	

two sign changes in the first column of the Routh table $\Rightarrow$ feedback system is unstable with two poles in the right half plane

b) $s(s-2)+5(s+1) = s^2+3s+5$ $\rightarrow$ all coefficients are positive in a second order polynomial $\Rightarrow$ feedback system is Stable.

c) $s(s^2+6s+10)(s+5) + 0.5(s+10)(s-2)^2 = s^4 + 11s^3 + 40s^2 + 50s + 0.5(s+10)(s^2-4s+4)$

$= s^4 + 10.5s^3 + 37s^2 + 32s + 20$ *

s^4	1	37	20
s^3	10.5	32	0
s^2	$\left(37 - \frac{32}{10.5}\right)$	20	0
s^1	x	0	
s^0	20		

$$x = \left(\left(37 - \frac{32}{10.5}\right) \times 32 - 20 \times 10.5\right) \Big/ \left(37 - \frac{32}{10.5}\right)$$

$$x \geq \frac{33 \times 32 - 20 \times 10.5}{\left(37 - \frac{32}{10.5}\right)} > 0$$

feedback system is stable.

2-b) System (a) is type zero with $k_P = 10$. $k_v =,0$ $k_a=0$,
And the steady state errors for unit step, ramp, and acceleration inputs are respectively 1/11, ∞, ∞.
System (b) is type one with $k_P = \infty$. $k_v =,-2.5$ $k_a=0$,
And the steady state errors for unit step, ramp, and acceleration inputs are respectively 0, -0.4, ∞.
System (c) is type one with $k_P = \infty$. $k_v =,0.4$, $k_a=0$,
And the steady state errors for unit step, ramp, and acceleration inputs are respectively 0, 2.5, ∞.

3-a) Consider a unity gain feedback control system. The plant transfer function is G(s)=1/(s^2+5s+6). Let the controller be of the form C(s) =K(s+z)/(s+p). Design the controller (ie choose K, z, p>0) so that the closed loop system has poles at $-1 \pm j$

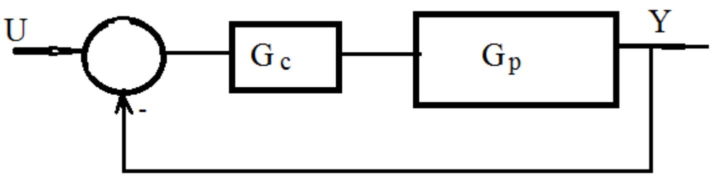


Figure 3

The open loop transfer function is given by $C(s)G(s) = \dfrac{K(s+z)}{(s+p)(s^2+5s+6)}$

The characteristic equation is given by $1 + \dfrac{K(s+z)}{(s+p)(s^2+5s+6)} = 0$ which is reduced to

$(s+p)(s^2+5s+6) + K(s+z) = 0$

$$s^3 + (5+p)s^2 + (6+5p+K)s + (Kz+6p) = 0$$

The function is devisible by (s+1-i)(s+1-i) i.e devisible by s^2+2s+2

i.e $s^3 + (5+p)s^2 + (6+5p+K)s + (Kz+6p) = (s^2+2s+2)(s+a) = 0$

$$(s^2+2s+2)(s+a) = 0$$

$$(s^2+2s+2)(s+a) = s^3 + (2+a)s^2 + (2+2a)s + 2a = 0$$

Comparing the coefficients

$$5 + p = 2 + a \Rightarrow \because p \succ 0 \Rightarrow a \succ 3$$

$$6+5(a-3)+k=2+2a$$

$$k=11-3a \Rightarrow a \prec \frac{11}{3}$$

$$kz+6p=2a \rightarrow kz+6(a-3)=2a$$

$$z(11-3a)=18-4a \Rightarrow a \prec \frac{11}{3} \Rightarrow z \succ 0$$

Multiplying the coefficient of s^2 by 2 and subtract the coefficient of s

$$4-3p-k=2 \rightarrow k+3p=2$$

$$0 \prec p \prec \frac{2}{3}, 0 \prec k \prec 2$$

$$kz=2a-6p \Rightarrow 2 \prec kz \prec \frac{22}{3} \quad \therefore z \succ 1$$

$$\therefore 0 \prec p \prec \frac{2}{3}, 0 \prec k \prec 2, z \succ 1$$

3-b) For the control system shown in figure 3, sketch the root locus for the following three cases, indicate its direction, where K = 0, where K = ∞, and if they exist, find asymptotes.

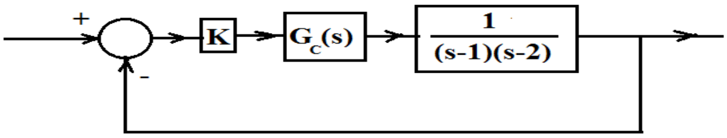
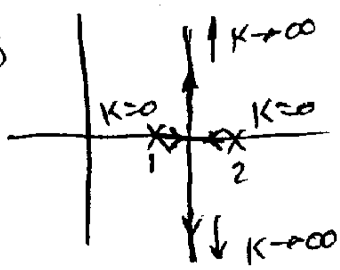


Figure 4

(i) **$G_c(s)$ = 1. , (ii) $G_c(s)$ = s+1 (PD compensation), (iii)$G_c(s)$ =1+1/s (PI compensation).**

For the appropriate choice of compensator, use root locus and Routh Hurwitz techniques to find the range of K for an under damped response

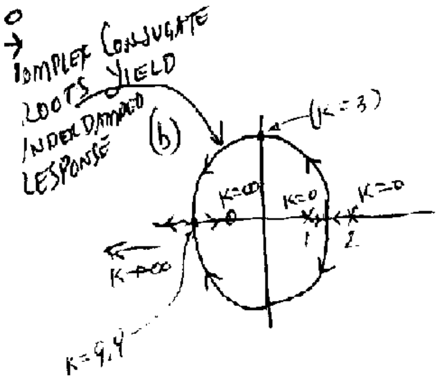
(i) $\frac{K}{(s-1)(s-2)}$



$\sigma_a = \frac{+1+2}{2} = \frac{3}{2}$

$\theta = \frac{180}{2} r = \mp 90^\circ$

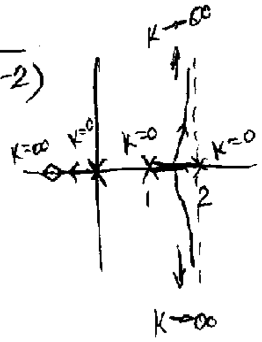
(ii) $\frac{K(s+1)}{(s-1)(s-2)}$



$\sigma_a = \frac{1+2-(-1)}{1} = 4$

$\theta = \frac{180}{1} r = 180^\circ$

(iii) $\frac{K(s+1)}{s(s-1)(s-2)}$



$\sigma_a = \frac{1+2-(-1)}{2} = 2$

$\theta = \frac{180}{2} r = \pm 90^\circ$

Because of the root locus typically approach the asymptotes as in (iii) , but may also lie on the asymptotes as in (i) and (ii)

$1 + \frac{K(s+1)}{(s-1)(s-2)} = 0 \Rightarrow s^2 + (K-3)s + K+2 \Rightarrow K = 3$ FOR MARGINAL STABILITY

$K = \frac{-(s-1)(s-2)}{s+1} \qquad \frac{dK}{ds} = 0 \Rightarrow s^2 + 2s - 5 = 0 \Rightarrow s = 1.45, -3.45$

$K\Big|_{s=-3.45} = -\frac{(-3.45-1)(-3.45-2)}{-3.45+1} = 9.9$

$3 < K < 9.9$ FOR UNDERDAMPED RESPONSE

4-a) Bode Plots of a stable plant $G_P(s)$ are shown in Figure 4 below. Design a proportional controller $G_c(s) = K$, so that the steady state error for a unit step input is as small as possible, and the gain margin of the feedback system is greater or equal to 5 db.

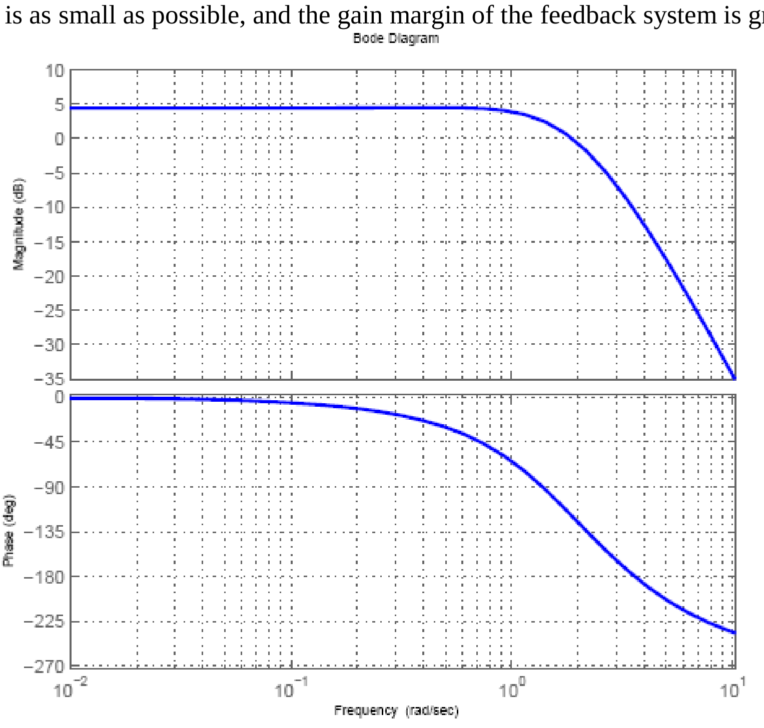
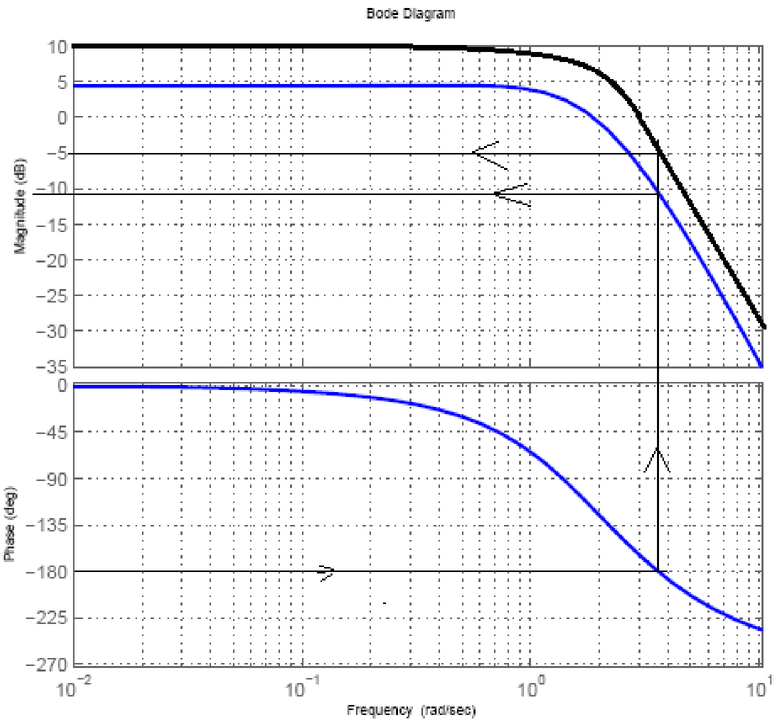


Figure 5

The proportional controller does not chsnge the phase but it does change then gain only

We can shift the Bode plot representing the gain 6db and keep a gain margin of at least 5db as shown in figure below



As ω tends to zero the gain approaches 10

$\therefore 10 = 20\log k_p \Rightarrow k_p = 3.16$

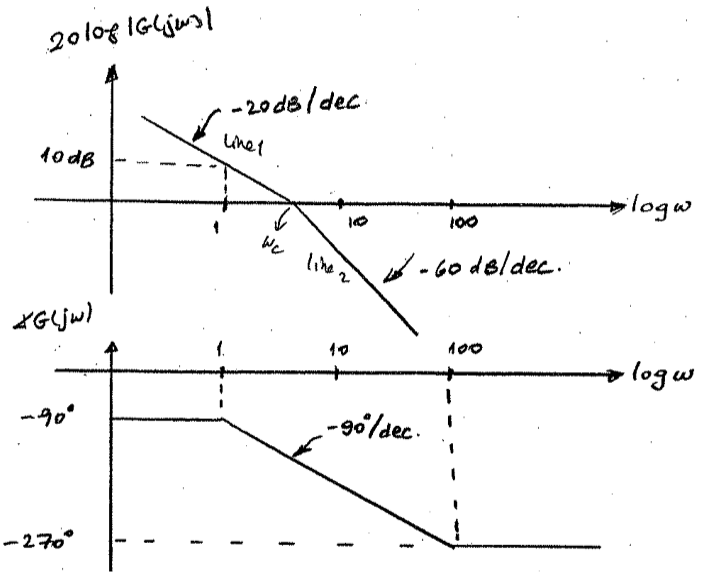
Before the controller $k_p = 10^{0.2} = 1.585$

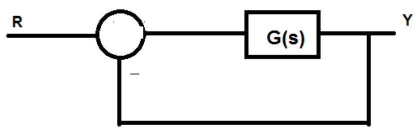
The steady state error for unit input $e_{ss} = \dfrac{1}{1+k_p}$ as the system is zero type so

The steady state error is reduced from approximately 0.4 to approximately 0.25

4-b) Consider the following system where G(s) is a transfer function. Asymptotic Bode plots of G(s) is given in figure 5 below. For calculations, you may use these Asymptotic plots.

ii) Find the gain margin of the system.
iii) Find the phase margin of the system.
iv) Find the transfer function of the system. .





line 1 $\Rightarrow$ $20\log K - 20\log \omega$ $\Rightarrow$ $\omega = 1 \Rightarrow 10\,dB \Rightarrow 20\log K = 10$ $\Rightarrow$ $\boxed{K = \sqrt{10}}$

$\Rightarrow$ $20\log K - 20\log \omega_c = 0$ $\Rightarrow$ $\boxed{\omega_c = K = \sqrt{10}}$

line 2 $\Rightarrow$ $A - 60\log \omega$ $= 0$ at $\omega_c = \sqrt{10}$ $\Rightarrow$ $A - 60\log\sqrt{10} = 0$ $\Rightarrow$ $A = 30$ dB. 05

i) at $\omega_c = \sqrt{10}$ $\Rightarrow$ $\angle = -90 - 90\log \omega_c = -90 - 45^\circ = -135^\circ$ $\Rightarrow$ $\boxed{PM = 180 - 135 = 45^\circ}$

ii) $90 - 90\log \omega_0 = -180$ $\Rightarrow$ $\omega_0 = 10$ rad/sec. $\Rightarrow$ $30 - 60\log 10 = -30\,dB$

$\Rightarrow$ $\boxed{GM = 30\,dB}$ 05

iii) $G(s) = \dfrac{K}{s\left(\frac{s}{\omega_c}+1\right)^2} = \dfrac{\sqrt{10}}{s\left(\frac{s}{\sqrt{10}}+1\right)^2} = \dfrac{3.16}{s\,(0.32s+1)^2}$ 05

GOOD LUCK